Test Booklet
High School Mathematics Competition 2019
Conducted by Lousing Chaphu, Thoubal
(*under the aegis of Private Schools' Development Council, Thoubal*)

BOOKLET CODE
C

Time: 2 hrs. **Full Marks: 100.**

INSTRUCTIONS

1. Write your name, roll number and centre name at the bottom of this page immediately after receiving this test booklet.
2. This test booklet contains 50 (***fifty***) Multiple Choice Questions (MCQs). Each question carries 1 mark in **PART A** (questions from 1 to 20), 2 marks in **PART B** (questions from 21 to 40) and 4 marks in **PART C** (questions from 41 to 50).
3. There shall be ***negative marking*** for each wrong answer, and ***a penalty of*** 25% ***of the mark allotted*** to the corresponding question shall be deducted. If nothing is marked for a question, no penalty shall be given.
4. For each question, there are four alternatives out of which ***only one is correct***. You have to find the correct answer and darken the letter corresponding to the answer only on the answer sheet.
5. Answer sheet has been provided separately. Before you start filling up your particulars, please ensure that this test booklet contains ***11 printed pages***.
6. Write your ***name***, ***roll number*** and ***booklet code of this test booklet*** on the answer sheet in the appropriate spaces. Also put ***your signature*** in the space provided.
7. You must darken the appropriate circles related to roll number and booklet code on the answer sheet using ***black or blue ball point pen*** only. It is also the sole responsibility of the candidate to meticulously follow the instructions given on the answer sheet.
8. Candidates found copying or resorting to any unfair means are liable to be disqualified from this examination.
9. Mobile phones or programmable calculators or any other electronic gadgets are strictly prohibited inside the examination hall.
10. Candidates are allowed to carry their test booklets at the end of the examination.

Name ______________________________

Roll number ______________________________

Centre name ______________________________

NOTATIONS

$\mathbb{N}$	the natural numbers $= \{1, 2, 3, \ldots\}$
$\mathbb{Z}$	the integers $= \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
$\mathbb{R}$	the real numbers

$\|x\|$	the absolute value of x
$\text{HCF}(a, b)$	the HCF of a and b
(x, y)	the coordinates of a point
$\not<$	not less than
$\not>$	not greater than

Unless otherwise stated, use $\pi = \dfrac{22}{7}$.

PART A

(Each question carries 1 mark.)

1. The more than ogive curve and the less than ogive curve of a frequency distribution intersect each other at the point $(20, 50)$. Then

(A) the mean is 20 and the size of the population is 50.

(B) the mean is 20 and the size of the population is 100.

(C) the median is 20 and the size of the population is 50.

(D) the median is 20 and the size of the population is 100.

2. If the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) lie on the same line, then

(A) $x_1(y_2 + y_3) + x_2(y_3 + y_1) + x_3(y_1 + y_2) = 0$.

(B) $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$.

(C) $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_2 - y_1) = 0$.

(D) $x_1(y_2 - y_3) + x_2(y_1 - y_3) + x_3(y_1 - y_2) = 0$.

3. If a die is thrown at random, then the probability of getting a prime is

(A) 0.2. **(B)** 0.3. **(C)** 0.5. **(D)** 0.6.

4. If $2x^2 + 3x + b = 0$ has equal roots, then the value of b is given by

(A) $b = -\frac{9}{4}$. **(B)** $b = \frac{9}{4}$. **(C)** $b = \frac{9}{8}$. **(D)** $b = -\frac{9}{8}$.

5. The quadratic equation whose roots are $-\alpha$ and $-\beta$ is

(A) $x^2 - (\alpha + \beta)x - \alpha\beta = 0$. **(B)** $x^2 + (\alpha + \beta)x - \alpha\beta = 0$.

(C) $x^2 + (\alpha + \beta)x + \alpha\beta = 0$. **(D)** $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

6. If the circumference of a circle exceeds the diameter by 210 cm, then the area of the circle is

(A) 7546 m^2. **(B)** 754.6 m^2. **(C)** 7.546 m^2. **(D)** 0.7546 m^2.

7. The sum S_n of the first n terms of an AP with first term a and common difference d is given by

(A) $S_n = \frac{n}{2}\{a + (n-1)d\}$. **(B)** $S_n = \frac{n}{2}\{a + (n+1)d\}$.

(C) $S_n = \frac{n}{2}\{2a + (n+1)d\}$. **(D)** $S_n = \frac{n}{2}\{2a + (n-1)d\}$.

8. Every prime greater than 3 is of the form

(A) $5k \pm 1$ for some integer k. **(B)** $6k \pm 1$ for some integer k.

(C) $7k \pm 1$ for some integer k. **(D)** $8k \pm 1$ for some integer k.

9. The value of $\tan 48^\circ \tan 23^\circ \sin 30^\circ \tan 42^\circ \tan 67^\circ$ is

(A) $\frac{1}{2}$. **(B)** $\frac{1}{\sqrt{2}}$. **(C)** 1. **(D)** $\frac{\sqrt{3}}{2}$.

10. Two triangles ABC and DEF are such that $AB = 5$ cm, $AC = 4$ cm and $DF = 36$ cm. If $\triangle ABC \sim \triangle DEF$, then DE is

(A) 60 cm. **(B)** 55 cm. **(C)** 50 cm. **(D)** 45 cm.

11. If a man goes 9 m due west and then 40 m due north, then his distance from the starting point is

(A) 49 m. **(B)** 43 m. **(C)** 42 m. **(D)** 41 m.

12. If a man stands at a point nearby a circular pond where the minimum distance of the point from the rim of the pond is 4 m, and the tangential distance of the point from the rim of the pond is 12 m, then the radius of the pond is

(A) 4 m. **(B)** 8 m. **(C)** 16 m. **(D)** 32 m.

13. If r and h are respectively the radius of the base and the height of a right circular cylinder, then the volume of the cylinder is

(A) $2\pi r^2 h$. **(B)** $\frac{1}{3}\pi r^2 h$. **(C)** $\frac{4}{3}\pi r^2 h$. **(D)** $\pi r^2 h$.

14. For a grouped frequency distribution, the mode is given by (the symbols have their usual meaning)

(A) $\text{mode} = l + \frac{f_m - f_1}{2f_m + f_1 - f_2} \times h$. **(B)** $\text{mode} = l + \frac{f_m - f_1}{2f_m - f_1 + f_2} \times h$.

(C) $\text{mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$. **(D)** $\text{mode} = l + \frac{f_m - f_1}{2f_m + f_1 + f_2} \times h$.

15. Is it possible to bend a wire of length l cm to form the legs (non-hypotenuse sides) of a right angled triangle with area 20 cm^2?

(A) Possible if $l < 11$. **(B)** Impossible if $l = 11$.

(C) Possible if $l = 12$. **(D)** Impossible if $l > 12$.

16. If A and B are two equally likely events, then

(A) $P(A) = \frac{1}{2}$.
(B) $P(A) = P(B)$.
(C) $P(AB) = P(A)P(B)$.
(D) $P(A) + P(B) = 1$.

17. If $a = bq + r$, where $a, b, q, r \in \mathbb{Z}$, then

(A) $\text{HCF}(a, b) = \text{HCF}(a, q)$.
(B) $\text{HCF}(a, b) = \text{HCF}(a, r)$.
(C) $\text{HCF}(a, b) = \text{HCF}(q, r)$.
(D) $\text{HCF}(a, b) = \text{HCF}(b, r)$.

18. Four years ago a father was nine times as old as his son, and 8 years hence the father's age will be three times the son's age. If x and y are the present ages of the father and the son respectively, then

(A) $x - 9y + 32 = 0$ and $x - 3y - 16 = 0$.
(B) $x + 9y - 32 = 0$ and $x - 3y - 16 = 0$.
(C) $x - 9y + 32 = 0$ and $x + 3y - 16 = 0$.
(D) $x + 9y - 32 = 0$ and $x + 3y - 16 = 0$.

19. Let A, B and C be the points (e, π), $(2e, 3\pi)$ and $(3e, 5\pi)$ respectively. Then

(A) A, B and C are collinear.
(B) the area of $\triangle ABC$ is $e\pi$.
(C) the area of $\triangle ABC$ is $2e\pi$.
(D) the area of $\triangle ABC$ is $3e\pi$.

20. If $x + 1$ is a factor of $kx^5 - x^4 + 3x^2 + 2$, then the value of k is

(A) 2. **(B)** 4. **(C)** -2. **(D)** -4.

PART B

(Each question carries 2 marks.)

21. The coordinates of the point which divides the line segment joining the points $(6, 1)$ and $(-4, 6)$ in the ratio $3 : 2$ internally are

(A) $(0, 5)$. **(B)** $(0, 4)$. **(C)** $(2, 4)$. **(D)** $(2, 5)$.

22. For any $x, y \in \mathbb{R}$,

(A) $|x + y| \leq |x| + |y|$ and $|x - y| \leq ||x| - |y||$.
(B) $|x + y| \geq |x| + |y|$ and $|x - y| \not< ||x| - |y||$.
(C) $|x + y| \not> |x| + |y|$ and $|x - y| \geq ||x| - |y||$.
(D) $|x + y| \not< |x| + |y|$ and $|x - y| \not> ||x| - |y||$.

23. A bridge over a river makes an angle of 45° with the river bank. If the length of the bridge over the river is 200 m, then the width of the river is

(A) 100 m. **(B)** $100\sqrt{2}$ m. **(C)** $100\sqrt{3}$ m. **(D)** 200 m.

24. When $x^5 - x^3 + kx^2 + 4x + 2$ is divided by $x^2 + 2x + 2$, the remainder is $-4x - 4$. Then the value of k must be

(A) -1. **(B)** 1. **(C)** -2. **(D)** 2.

25. The expression $a^2(b+c) + b^2(c+a) + c^2(a+b) + 3abc$ factorises to

(A) $(a+b+c)(ab+bc+ca)$.

(B) $(a+b+c)(a^2+b^2+c^2)$.

(C) $(a+b)(b+c)(c+a)$.

(D) $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$.

26. In $\triangle ABC$, a line parallel to BC cuts AB and AC at D and E respectively such that $AD = x$ cm, $DB = 5$ cm, $AE = 2$ cm and $EC = x - 3$ cm. Then the value of x is

(A) 5. **(B)** 7. **(C)** 10. **(D)** 15.

27. There are five men and three ladies in a council. If two council members are selected at random for a committee, then the probability that the selected members have different sex is

(A) $\frac{15}{56}$. **(B)** $\frac{20}{56}$. **(C)** $\frac{30}{56}$. **(D)** $\frac{40}{56}$.

28. Consider the following statements.

1. The coordinates of the mid-point of the line segment joining (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

2. The coordinates of the centroid of the triangle with the vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

Then

(A) 1 is true but 2 is false. **(B)** 1 is false but 2 is true.

(C) both 1 and 2 are true. **(D)** both 1 and 2 are false.

29. The values of k and l for which the pair of linear equations $3x + y = 1$ and $(2k-1)x + (k-1)y = l$ has no solution are given by

(A) $k \neq 2, l \neq 1$. **(B)** $k \neq 2, l = 1$. **(C)** $k = 2, l \neq 1$. **(D)** $k = 2, l = 1$.

30. Let $p(x)$ be a polynomial such that when divided by $x - 2$ it leaves the remainder 3 and when divided by $x - 3$ it leaves the remainder 2. Then the remainder when it is divided by $(x-2)(x-3)$ is

(A) $x + 5$. **(B)** $-x + 5$. **(C)** $x - 5$. **(D)** $-x - 5$.

31. Consider the following statements.

1. The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the other two sides.

2. The ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides.

Then

(A) 1 is true but 2 is false. **(B)** 1 is false but 2 is true.

(C) both 1 and 2 are true. **(D)** both 1 and 2 are false.

32. If r_1 and r_2 $(r_1 > r_2)$ are the radii of the bases and h is the height of a frustum, then the volume V and the curved surface area S of the frustum are respectively given by

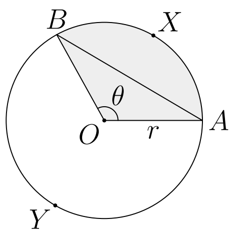
(A) $V = \frac{1}{3}\pi h(r_1^2 + r_1 r_2 + r_2^2)$ and $S = \pi l(r_1 + r_2)$.

(B) $V = \frac{1}{3}\pi h(r_1^2 + r_1 r_2 + r_2^2)$ and $S = 2\pi l(r_1 + r_2)$.

(C) $V = \frac{4}{3}\pi h(r_1^2 + r_1 r_2 + r_2^2)$ and $S = \pi l(r_1 + r_2)$.

(D) $V = \frac{4}{3}\pi h(r_1^2 + r_1 r_2 + r_2^2)$ and $S = 2\pi l(r_1 + r_2)$.

33. In the following figure, $\overline{AB}$ is a chord of a circle with centre O.



Consider the following statements.

1. The length of the arc $\widehat{AXB}$ is $2r\theta$, where θ is measured in radians.

2. The area of the minor segment AXB is $\frac{r^2}{2}\left(\frac{\theta\pi}{180} - \sin\theta\right)$, where θ is measured in degrees.

Then

(A) 1 is true but 2 is false. **(B)** 1 is false but 2 is true.

(C) both 1 and 2 are true. **(D)** both 1 and 2 are false.

34. If θ is an acute angle, then

(A) $\sin(90^\circ - \theta) = \cos\theta$, $\tan(90^\circ - \theta) = \sec\theta$.

(B) $\cos(90^\circ - \theta) = \sec\theta$, $\cot(90^\circ - \theta) = \tan\theta$.

(C) $\tan(90^\circ - \theta) = \cot\theta$, $\sec(90^\circ - \theta) = \cos\theta$.

(D) $\sin(90^\circ - \theta) = \cos\theta$, $\tan(90^\circ - \theta) = \cot\theta$.

35. Consider the two APs $3, 7, 11, \ldots, 407$ and $2, 9, 16, \ldots, 709$. The number of common terms of these APs is

(A) 12. **(B)** 13. **(C)** 14. **(D)** 15.

36. Let $ax^2 + bx + c = 0$, where $a, b, c \in \mathbb{R}$ and $a \neq 0$. Consider the following statements.

1. The roots of the above quadratic equation are real if $b^2 - 4ac \geq 0$.

2. If $a + b + c = 0$, then the roots of $ax^2 + bx + c = 0$ are 1 and $-\frac{c}{a}$.

Then

(A) 1 is true but 2 is false. **(B)** 2 is true but 1 is false.

(C) both 1 and 2 are true. **(D)** both 1 and 2 are false.

37. For a frequency distribution of marks obtained by 2000 students in a competitive examination, the mean mark is 50.51 and the 90th percentile $P_{90} = 70.34$. Consider the following statements.

1. About half of the students scored less than 50.51 marks and the other half scored more than 50.51 marks.

2. About 1800 students scored below 70.34 marks and about 200 students scored above 70.34 marks.

Then

(A) 1 is true but 2 is false.

(B) 1 is false but 2 is true.

(C) both 1 and 2 are true.

(D) both 1 and 2 are false.

38. Consider the following statements.

1. Cancellation law for multiplication: If x, y and z are real numbers such that $xy = xz$, then $y = z$.

2. The set $\mathbb{N}$ of natural numbers contains an additive identity as well as a multiplicative identity.

Then

(A) 1 is true but 2 is false.

(B) 1 is false but 2 is true.

(C) both 1 and 2 are true.

(D) both 1 and 2 are false.

39. The least multiple of 11 which when divided by 7, 8 and 12 leaves the same remainder 4 in each case is

(A) 550. **(B)** 2860. **(C)** 396. **(D)** 1012.

40. Four cows are tethered with ropes of equal length at the four corners of a square field of side 28 m so that consecutive cows can just reach each other. The area of the field that will remain ungrazed is

(A) 630 m^2. **(B)** 616 m^2. **(C)** 476 m^2. **(D)** 168 m^2.

PART C

(Each question carries 4 marks.)

41. If $x + y + z = 5$, $x^2 + y^2 + z^2 = 113$ and $xyz = 67$, then the values of $(x + y)(y + z)(z + x)$ and $(x + y + z)^3 - x^3 - y^3 - z^3 + 3xyz$ are respectively

(A) -287 and -660.

(B) 287 and 660.

(C) -287 and 660.

(D) 287 and -660.

42. A spherical balloon of radius r metres subtends an angle α at the eye of an observer, while the angle of elevation of its centre from the eye of the observer is β. Then the vertical distance of the centre of the balloon in metres from the horizontal through the eye of the observer is

(A) $r \cos\beta \sec\left(\frac{\alpha}{2}\right)$.

(B) $r \cos\beta \operatorname{cosec}\left(\frac{\alpha}{2}\right)$.

(C) $r \sin\beta \sec\left(\frac{\alpha}{2}\right)$.

(D) $r \sin\beta \operatorname{cosec}\left(\frac{\alpha}{2}\right)$.

43. To construct a $\triangle AB'C'$ similar to a given triangle ABC with a scale factor of $\frac{5}{7}$, first a ray AX is drawn such that $\angle BAX$ is an acute angle and X is on the opposite side of C with respect to AB. Secondly, we mark n (large enough) points P_i's $(i = 1, 2, \ldots, n)$ on AX such that $AP_1 = P_iP_{i+1}$ for all $i = 1, 2, \ldots, n-1$. Thirdly, we join P_lB and draw P_mB' parallel to P_lB meeting AB or AB produced at B'. Lastly, we draw $B'C'$ parallel to BC meeting AC or AC produced at C' to get the required triangle. Then we may take

(A) $l = 12$ and $m = 5$.

(B) $l = 12$ and $m = 7$.

(C) $l = 5$ and $m = 7$.

(D) $l = 7$ and $m = 5$.

44. The internal and external radii of a hollow metallic sphere are 3 cm and 5 cm respectively. If the sphere is melted and recast into a solid cylinder of height $\dfrac{2}{3}$ cm, then the curved surface area of the cylinder is

(A) 176 cm^2.

(B) $\dfrac{176}{3}$ cm^2.

(C) $\dfrac{88}{3}$ cm^2.

(D) $\dfrac{44}{3}$ cm^2.

45. For $0 < \theta < 90°$, consider the following statements.

1. $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$.

2. $\dfrac{1-\sin\theta}{1+\sin\theta} = (\sec\theta + \tan\theta)^2$.

3. $\tan^2\theta = \sec^2\theta + 1$.

4. $\operatorname{cosec}^2\theta = \cot^2\theta + 1$.

Then

(A) only one statement is true.

(B) only two statements are true.

(C) only three statements are true.

(D) all the statements are true.

46. Consider the following statements regarding the lines $x + y + 1 = 0$ and $ax + by + 1 = 0$, where a and b are constants such that $a^2 + b^2 \neq 0$.

1. The lines intersect at exactly one point, namely $\left(\dfrac{b-1}{a-b}, \dfrac{1-a}{a-b}\right)$ if and only if $a \neq b$.

2. The lines are coincident if and only if $a = b = 1$.

3. The lines are parallel if and only if $a = b \neq 1$.

Then

(A) 2, 3 are true but 1 is false.

(B) 1, 3 are true but 2 is false.

(C) 1, 2 are true but 3 is false.

(D) all are true.

47. Two circles with radii a and b touch each other externally. Let c be the radius of the circle that touches these two circles externally as well as a common tangent to the two circles. Then

(A) $\frac{1}{8c^2} = \frac{1}{a^2} + \frac{1}{b^2}$. **(B)** $\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{2a}} + \frac{1}{\sqrt{2b}}$.

(C) $\frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$. **(D)** $\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}$.

48. The ratio of the sum of the first m terms to that of the first n terms of an AP with first term a and common difference d is $m^2 : n^2$. Then the ratio of the mth term to the nth term of the AP is

(A) $m : n$. **(B)** $m^2 : n^2$.

(C) $(2m - 1) : (2n - 1)$. **(D)** $(m^2 - n) : (n^2 - m)$.

49. Consider the following distribution.

Class	Frequency
0 – 10	8
10 – 20	5
20 – 30	2
30 – 40	2
40 – 50	3

The mean and the median of the distribution are respectively

(A) 18.5 and 14. **(B)** 18.5 and 12. **(C)** 18 and 14. **(D)** 18 and 12.

50. Consider the following statements.

1. If ABC and DEF are two triangles such that $\angle A = \angle B$, $\angle D = \angle E$ and $\angle C = \angle F$, then $\triangle ABC \sim \triangle DEF$.

2. If ABC and DEF are two triangles such that $\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

3. All similar figures are congruent.

4. All rectangles are similar.

Then

(A) only one statement is true. **(B)** only two statements are true.

(C) only three statements are true. **(D)** all the statements are true.

Answer Key: C
High School Mathematics Competition 2019
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1. (D)
2. (B)
3. (C)
4. (C)
5. (C)
6. (D)
7. (D)
8. (B)
9. (A)
10. (D)
11. (D)
12. (C)
13. (D)
14. (C)
15. (B)
16. (B)
17. (D)
18. (A)
19. (A)
20. (B)
21. (B)
22. (C)
23. (B)
24. (B)
25. (A)
26. (A)
27. (C)
28. (C)
29. (C)
30. (B)
31. (A)
32. (A)
33. (B)
34. (D)
35. (C)
36. (A)
37. (B)
38. (D)
39. (D)
40. (D)
41. (A)
42. (D)
43. (D)
44. (B)
45. (B)
46. (D)
47. (D)
48. (C)
49. (A)
50. (B)