# Mathematics X

**Previous Years' Questions and Answers**

Based on the syllabus prescribed by the BSEM
For Class X

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Supplement to Second Edition

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# MATHEMATICS X

PREVIOUS YEARS' QUESTIONS AND ANSWERS



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# Contents

# Design of Question Paper

Time: 3 hours. Full Marks: 80.

1. Weightage to objectives:

| Objectives | K | U | A | S | Total |
|---|---|---|---|---|---|
| Percentage of marks | 37 | 45 | 12 | 6 | 100 |
| Marks | 30 | 36 | 9 | 5 | 80 |

(K = Knowledge, U = Understanding, A = Application, S = Skill.)

2. Weightage to forms of questions:

| Forms of questions | LA | $SA_1$ | $SA_2$ | $SA_3$ | VSA | O | Total |
|---|---|---|---|---|---|---|---|
| Number of questions | 5 | 3 | 6 | 5 | 8 | 5 | 32 |
| Marks allotted | 27 | 12 | 18 | 10 | 8 | 5 | 80 |
| Estimated time (min) | 70 | 33 | 36 | 20 | 13 | 8 | 180 |

(LA = Long Answer, SA = Short Answer, VSA = Very Short Answer, O = Objective.)

3. Weightage of content:

| Unit | Name of the unit | Marks |
|---|---|---|
| I | Number System, Polynomials and Factorisation | 15 |
| II | Pair of Linear Equations in Two Variables, Quadratic Equations and AP | 15 |
| III | Triangles, Circles and Construction | 15 |
| IV | Trigonometric Ratios, Height and Distances and Coordiante Geometry | 15 |
| V | Mensuration | 10 |
| VI | Statistics and Probability | 10 |
| | Total | 80 |

4. Scheme of section: Nil.

5. Scheme of option: Internal option must be given in essay/long answer type questions testing the same objective.

6. Difficulty level: 20% difficult, 60% average, 20% easy.

# Chapter 1

# High School Leaving Certificate Examination

## Question Papers

**2010**
**Mathematics**
*Time : Three hours*
*For Question Nos. 1 to 5, write the letter corresponding to the correct answer*
*The figures in the right hand margin indicate the full marks for the questions.*

**1.** Let $p(x)$ be a polynomial of degree $\geq 1$ and $a$ be any real number. If $p(x)$ is divided by $x-a$, then the remainder is 1

**(A)** $-p(-a)$. **(B)** $-p(a)$. **(C)** $p(-a)$. **(D)** $p(a)$.

**2.** The value of $\cot 35° \cot 55°$ is 1

**(A)** 0. **(B)** 1. **(C)** $\sqrt{3}$. **(D)** $\frac{1}{\sqrt{3}}$.

**3.** The length of the shadow of a tower is 20 m when the altitude of the sun is 60°. The height of the tower in metres, is 1

**(A)** $20\sqrt{3}$. **(B)** $\frac{20\sqrt{3}}{3}$. **(C)** $\frac{40\sqrt{3}}{3}$. **(D)** $40\sqrt{3}$.

**4.** If the points $(a, 0)$, $(2, 3)$ and $(0, 2)$ are collinear, then the value of $a$ is

**(A)** 1. **(B)** −1. **(C)** −4. **(D)** 4. 1

**5.** The area of a sector of a circle with radius $r$ and sectorial angle $\theta$ measured in degree, is 1

**(A)** $\frac{\theta\pi r^2}{360}$. **(B)** $\frac{\theta\pi r^2}{180}$. **(C)** $\frac{\theta\pi r}{360}$. **(D)** $\frac{\theta\pi r}{180}$.

**6.** Find the quotient when $x^3 - 1$ is divided by $x^2 + x + 1$. 1

**7.** When is a pair of linear equations said to be a dependent pair? 1

**8.** For what values of $k$ does the pair of equations $2x + 3y + 6 = 0$ and $4x + ky + 12 = 0$ have unique solution? 1

**9.** What is meant by the discriminant of a quadratic equation $ax^2+bx+c = 0$? 1

**10.** Define an arithmetic progression. 1

**11.** Write the statement of Pythagoras theorem. 1

**12.** Write the coordinates of the mid-point of the line segment joining $(x_1, y_1)$ and $(x_2, y_2)$. 1

**13.** Find the curved surface area of a cylinder of radius 3 cm and height 7 cm. 1

**14.** Define the terms **(i)** sample space and **(ii)** event, associated with a random experiment. 1

**15.** If $x \in \mathbb{R}$, prove that $|-x| = |x|$. 2

**16.** Factorise $ab(a + b) + bc(b + c) + ca(c + a) + 3abc$. 2

**17.** Prove the formula $S_n = \frac{n}{2}[2a + (n - 1)d]$ for finding the sum of first $n$ terms of an AP. 2

**18.** The $n^{\text{th}}$ term of a sequence is given by $a_n = 3n + 2$. Show that the sequence is an AP. 2

**19.** The length of a tangent to a circle from a point which is at a distance of 5 cm from the centre of the circle is 4 cm. Find the radius of the circle. 2

**20.** State and prove factor theorem. 3

**21.** Express in the form $a^3+b^3+c^3-3abc$ and hence factorise $x^6+8x^3+27$. 3

**22.** Solve graphically: 3

$$3x + y = 9 \text{ and } 2x - 3y + 16 = 0.$$

**23.** Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. 3

**24.** Find the values of the trigonometric ratios of 45°. 3

**25.** Prove that 3

$$\frac{\sin\theta}{\sec\theta+1} + \frac{\sin\theta}{\sec\theta-1} = 2\cot\theta.$$

**26.** Find the least positive multiple of 17 which when divided by 6, 9 and 15 leaves the same remainder in each case. 4

**27.** Solve by the method of completing perfect square, the equation $ax^2 + bx + c = 0$, $a \neq 0$. 4

**28.** Prove that the coordinates of the point $R$ which divides the line joining $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $m : n$ are 4

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right).$$

*Or*

Show that the area of a $\triangle ABC$ with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is 4

$$\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|.$$

**29.** A solid metallic cone is 27 cm high and radius of its base is 16 cm. If it is melted and recast into a solid sphere, find the curved surface area of the sphere. 4

**30.** State and prove basic proportionality theorem. 5

*Or*

Prove that the internal bisector of an angle of a triangle divides the opposite site internally in the ratio of the other two sides. 5

**31.** Draw any line segment and divide it internally in the ratio 3 : 5. Write the steps of construction. 5

**32.** A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height $h$. At a point on the plane, the angle of elevation of the bottom of the flagstaff is $\alpha$ and that of the top is $\beta$. Find the height of the tower. 5

*Or*

The angle of elevation of a bird from the eye of a man on the bank of a pond is 30° and angle of depression of its reflection in the pond is 60°. Find the height of the bird above the pond if the eye is 1.5 m above the water level. 5

**33.** Two dice are thrown and the points on them are added together. Find which is more likely to happen that the sum is 7 and that the sum is 8. 5

**34.** A bucket is in the form of a frustum with a capacity of 45584 cm$^3$. If the radii of the top and bottom of the bucket are 28 cm and 21 cm respectively, find its height and surface area. 6

**35.** The following is the grouped data of the number of persons of various age groups in a hill village in a border area of Manipur. 6

| Age group | No. of persons |
|---|---|
| 0 – 10 | 51 |
| 10 – 20 | 55 |
| 20 – 30 | 78 |
| 30 – 40 | 75 |
| 40 – 50 | 62 |
| 50 – 60 | 47 |
| 60 – 70 | 23 |
| 70 – 80 | 7 |
| 80 – 90 | 2 |
| 90 – 100 | 0 |

Find the mean age and the median age of the inhabitants of the village.

## 2011

**1.** If $x-3$ is a factor of $x^2+\lambda x+3$, then the value of $\lambda$ is 1

**(A)** $-3$. **(B)** $3$. **(C)** $-4$. **(D)** $4$.

**2.** The expression $a^2(b+c)+b^2(c+a)+c^2(a+b)+2abc$ factorises to 1

**(A)** $(a+b)(b+c)(c+a)$. **(B)** $-(a-b)(b-c)(c-a)$.

**(C)** $3(a+b)(b+c)(c+a)$. **(D)** $(a+b+c)(ab+bc+ca)$.

**3.** The linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ form an inconsistent pair if 1

**(A)** $\frac{a_1}{a_2}=\frac{b_1}{b_2}$. **(B)** $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$.

**(C)** $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$. **(D)** $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$.

**4.** For what value of $k$ will the equation $x^2-6x+k=0$ have equal roots?

**(A)** $6$. **(B)** $9$. **(C)** $-6$. **(D)** $-9$. 1

**5.** The area of a circle of radius $r$ is 1

**(A)** $\pi r$. **(B)** $2\pi r$. **(C)** $\pi r^2$. **(D)** $2\pi r^2$.

**6.** When is an algebraic expression said to have cyclic factors? 1

**7.** When is a number $\alpha$ said to be a root of the quadratic equation $ax^2+bx+c=0$? 1

**8.** Find the sum of the following AP: $1, 3, 5, 7, \ldots$ to 10 terms. 1

**9.** When is a line said to be a tangent to a circle? 1

**10.** $PA$ and $PB$ are the tangent segments drawn from an external point $P$ to a circle with centre $O$. If $\angle AOB=130°$, at what angle are two tangents inclined to each other? 1

**11.** Write the coordinates of the mid-point of the line segment joining the points $(x_1, y_1)$ and $(x_2, y_2)$. 1

**12.** Prove that $\tan 40° \tan 45° \tan 50° = 1$. 1

**13.** Express the length of arc of a sector of a circle with radius $r$ and angle $\theta$ degrees, in terms of $r$ and $\theta$. 1

**14.** Prove that $|x|^2 = x^2$, for any real number $x$. 2

**15.** Prove that in an AP with first term $a$ and common difference $d$ the $n^{\text{th}}$ term is given by $a_n = a + (n-1)d$. 2

**16.** Form the quadratic equation whose roots are $3 + \sqrt{5}$ and $3 - \sqrt{5}$. 2

**17.** Construct a pair of tangents to a circle from an external point (traces of construction only). 2

**18.** In a right $\triangle ABC$, right angled at $B$, show that $\cos^2 A + \sin^2 A = 1$. 2

**19.** Show that every odd integer is of the form $4k + 1$ or $4k - 1$. 3

**20.** State and prove remainder theorem. 3

**21.** If $\alpha$ and $\beta$ are the roots of the quadratic equation $ax^2 + bx + c = 0$, then prove that $\alpha\beta = \dfrac{c}{a}$. 3

**22.** If $\tan\theta = \dfrac{a}{b}$, show that 3

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}.$$

**23.** Prove that the area of a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is 3

$$\frac{1}{2}\Big|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\Big|.$$

**24.** A solid metallic cylinder of height 24 cm and radius 3 cm is melted and recast into a cone of radius 6 cm. Find the height of the cone. 3

**25.** State Euclid's algorithm for finding the HCF of two given positive integers, stepwise. 4

**26.** A man invested Rs 36000, a part of it at 12% and the rest at 15% per annum simple interest. If he received a total annual interest of Rs 4890, how much did he invest at each rate? 4

**27.** Two dice are thrown. Find the probability that the sum of their points is 10. 4

**28.** State and prove SAS similarity theorem. 5

*Or*

Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. 5

**29.** A tower subtends an angle $\alpha$ at a point on the same level as the foot of the tower and from a second point $h$ metres above the first, the angle of depression of the foot of the tower is $\beta$. Find the height of the tower. 5

*Or*

The angles of depression of the top and bottom of a 8 m tall tree from the top of a tower are $45°$ and $60°$ respectively. Find the height of the tower. 5

**30.** A cone is cut into three parts by planes through the points of trisection of its altitude and parallel to the base. Prove that the volume of the parts are in the ratio $1 : 7 : 19$. 5

*Or*

A geyser is in the form of a cylinder with hemispherical ends. If the length of the cylindrical portion is 56 cm and the diameter of each hemispherical end is 18 cm, find the capacity of the geyser in litres. 5

**31.** Construct a triangle similar to a given triangle $ABC$ with its sides equal to $\frac{7}{4}$ of the corresponding sides of the $\triangle ABC$. Write the steps of construction. 6

**32.** Find the mean and the mode of the following distribution: 6

| Marks below: | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|
| No. of students: | 15 | 35 | 60 | 84 | 96 | 127 | 198 | 250 |

## 2012

**1.** The remainder when $x^3 - 2x^2 + 3x - 1$ divided by $x - 2$ is 1

**(A)** 5. **(B)** $-5$. **(C)** 23. **(D)** $-23$.

**2.** The sum of the first $n$ terms of an AP with first term $a$ and common difference $d$, is given by 1

**(A)** $S_n = n[a + (n-1)d]$. **(B)** $S_n = \frac{n}{2}[a + (n-1)d]$.

**(C)** $S_n = \frac{n}{2}[2a + (n-1)d]$. **(D)** $S_n = \frac{n}{2}[a + 2(n-1)d]$.

**3.** The quadratic equation $x^2 - x + 1 = 0$ has 1

**(A)** two unequal real roots. **(B)** two equal real roots.

**(C)** two irrational roots. **(D)** no real roots.

**4.** If $\theta$ is an acute angle such that $\cos\theta = \frac{3}{5}$, then the value of $\tan\theta$ is

**(A)** $\frac{4}{3}$. **(B)** $\frac{3}{4}$. **(C)** $\frac{4}{5}$. **(D)** $\frac{5}{4}$. 1

**5.** If $A$ and $\overline{A}$ are complementary events of each other, the value of $P(A) + P(\overline{A})$ is 1

**(A)** 0. **(B)** 1. **(C)** $\frac{1}{2}$. **(D)** 2.

**6.** Is there any $x \in \mathbb{R}$ such that $x^2$ is not positive? 1

**7.** Write all the possible values of the remainder when a number is divided by 3. 1

**8.** What is meant by the discriminant of a quadratic equation? 1

**9.** For what value of $k$ will the roots of the equation $2x^2 - 5x + k = 0$ be reciprocal of each other? 1

**10.** Write the statement of the converse of Pythagoras theorem. 1

**11.** How long is the radius of a circle circumscribing a rectangle of sides 8 cm and 6 cm? 1

**12.** Write the expression for the area of a sector of a circle with radius $r$ and angle $\theta$ (in degrees). 1

**13.** Define mutually exclusive events associated with a random experiment. 1

**14.** For any $x \in \mathbb{R}$, prove that $x \cdot 0 = 0$. 2

**15.** If $\alpha$ and $\beta$ are the roots of the quadratic equation $ax^2 + bx + c = 0$, show that $\alpha + \beta = -\dfrac{b}{a}$. 2

**16.** Form the quadratic equation whose roots are $4 + \sqrt{5}$ and $4 - \sqrt{5}$. 2

**17.** If $0^\circ < 6\theta < 90^\circ$ and $\cos 4\theta = \sin 6\theta$, find $\theta$ in degrees. 2

**18.** A ball is drawn at random from an urn containing 4 black and 5 red balls. Find the probability that the ball is red. 2

**19.** State and prove factor theorem. 3

**20.** By what numbers may 408 be divided so that the remainder is 23? 3

**21.** Solve graphically: 3

$$x + y = 5 \text{ and } 2x + 3y = 12.$$

**22.** Prove that the lengths of tangents drawn from an external point to a circle are equal. 3

**23.** Prove the identity: 3

$$\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}.$$

**24.** A toy is in the form of a cone mounted on a hemisphere with the same radius. The diameter of the base of the conical portion is 6 cm and its height is 4 cm. Determine the surface area of the toy. 3

**25.** Factorise $a^3 + b^3 + c^3 - 3abc$ or $2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4$. 4

**26.** A manufacturing company produced 600 cars in the third year and 700 cars in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production in the tenth year. 4

**27.** Prove that the coordinates of the point $R$ which divides the line joining $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $m : n$ are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right).$$ 4

**28.** Draw a line segment $AB$ and divide it in the ratio 4 : 3. Write the steps of construction. 5

**29.** An aeroplane when 3000 metres high, passes vertically above another at an instant when the angles of elevation at the observing point on the ground, are $60°$ and $45°$ respectively. How many metres is the one higher than the other? (Take $\sqrt{3} = 1.73$.) 5

*Or*

Two towers of the same height stand on the either side of a road 60 m wide. At a point on the road between the towers, the elevations of the towers are $60°$ and $30°$. Find the heights of the towers and position of the point. (Take $\sqrt{3} = 1.73$.) 5

**30.** A container made up of metal sheet is in the form of a frustum of a cone of height 20 cm with radii of its lower and upper ends as 10 cm and 25 cm respectively. Find the cost of milk which can fill the container at the rate of Rs 20 per litre and the cost of metal sheet used to make the container of it costs Rs 10 per 100 cm$^2$. (Take $\sqrt{3} = 1.73$.) 5

**31.** State and prove either AAA similarity theorem or SSS similarity theorem. 6

**32.** Find the mean and the median marks from the following frequency table: 6

| Marks above: | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| No. of students: | 80 | 77 | 72 | 65 | 55 | 43 | 28 | 16 | 10 | 8 | 0 |

## 2013

**1.** If $p(x)$ is a polynomial of degree $\geq 1$ and $a$ is any real number, then $x-a$ is a factor of $p(x)$ if and only if 1

**(A)** $p(a)=1$. **(B)** $p(a)=0$. **(C)** $p(-a)=0$. **(D)** $p(a)=a$.

**2.** The sum of the roots of the quadratic equation $ax^2+bx+c=0$ is equal to 1

**(A)** $-\frac{b}{a}$. **(B)** $\frac{b}{a}$. **(C)** $-\frac{c}{a}$. **(D)** $\frac{c}{a}$.

**3.** The number of two digit numbers which are divisible by 7 is 1

**(A)** 11. **(B)** 12. **(C)** 13. **(D)** 14.

**4.** If $\tan(2\theta+25^\circ)=\cot 3\theta$, then the value of $\theta$ is 1

**(A)** $10^\circ$. **(B)** $11^\circ$. **(C)** $13^\circ$. **(D)** $15^\circ$.

**5.** Area of a sector of a circle with radius $r$ and $\theta$ measured in degrees is

**(A)** $\frac{2\pi r\theta}{360}$. **(B)** $\frac{2\pi r^2\theta}{360}$. **(C)** $\frac{\pi r\theta^2}{360}$. **(D)** $\frac{\pi r^2\theta}{360}$. 1

**6.** What is meant by a cyclical expression? 1

**7.** Find the canonical decomposition of 2013. 1

**8.** State the nature of the roots of the quadratic equation $ax^2+bx+c=0$ when $b^2-4ac=0$. 1

**9.** Find the common difference of the AP whose $n^{\text{th}}$ term is $3n-2$. 1

**10.** Write the statement of Pythagoras theorem. 1

**11.** Write the formula for the volume of a frustum of a cone. 1

**12.** How long is an arc of a sector of a circle with radius 6 cm and angle $30^\circ$? (Take $\pi=3.14$.) 1

**13.** What is meant by a random experiment? 1

**14.** Show that any square number is of the form $4k$ or $4k+1$. 2

**15.** If $a$ be the first term and $d$ the common difference of an AP, then show that the $n^{\text{th}}$ term $a_n = a + (n-1)d$. 2

**16.** If one of the roots of the equation $x^2 - px + q = 0$ is twice the other, show that $2p^2 = 9q$. 2

**17.** Prove the identity: $\cos^2\theta - \sin^2\theta = \dfrac{\cot^2\theta - 1}{\cot^2\theta + 1}$. 2

**18.** A solid metallic cone is 12 cm high and radius of its base is 3 cm. It is melted and recast into a solid sphere. Find the radius of the sphere. 2

**19.** State and prove remainder theorem. 3.

**20.** Factorise $x(y^2 + z^2) + y(z^2 + x^2) + z(x^2 + y^2) + 3xyz$. 3

**21.** Solve graphically: $x + 2y = 7$, $2x - y = 4$. 3

**22.** Two concentric circles are of radii 6 cm and 10 cm. Find the length of the chord of the larger circle which touches the smaller circle. 3

**23.** Find the values of any two trigonometric ratios of $60°$. 3

**24.** A fair coin is tossed 3 times. Find the probability that head appears exactly twice. 3

**25.** If $x, y \in \mathbb{R}$ and $xy = 0$, then prove that $x = 0$ or $y = 0$. 4

*Or*

For real numbers $x, y, \delta$, where $\delta > 0$, prove that 4

$$|x - y| < \delta \iff y - \delta < x < y + \delta.$$

**26.** In a classroom, there are a number of benches. If 4 students sit on each bench, 5 benches are left vacant and if 3 students sit on each bench, 4 students are left standing. Find the number of benches and the number of students in the classroom. 4

**27.** Prove that the area of a triangle with vertices $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ is $\dfrac{1}{2}\Big|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\Big|$. 4

**28.** Construct a triangle similar to a given triangle $ABC$, with each sides equal to $\dfrac{7}{4}$ of the corresponding sides of the $\triangle ABC$. Write the steps of construction. $2 + 3$

**29.** A tower subtends an angle of 60° at a point on the same level as the foot of the tower and from a second point 10 metres above the first, the angle of depression of the foot of the tower is 30°. Find the height of the tower. 5

**30.** The radius and the height of a circular cone are 10 cm and 25 cm respectively. The area of cross-section of the cone by a plane parallel to its base is 154 $cm^2$. Find the distance of the plane from the base of the cone. (Take $\pi = \frac{22}{7}$.) 5

*Or*

A vessel is in the form of an inverted cone of height 6 cm and radius 4 cm. It is filled with water upto the rim. When lead shots each of which is a sphere of radius 0.2 cm are dropped into the vessel, one-tenth of the water flows out. Find the number of lead shots dropped into the vessel. 5

**31.** State and prove basic proportionality theorem. 6

*Or*

If a perpendicular is drawn from the vertex of a right angle of a right triangle to the hypotenuse, prove that the triangles on each side of the perpendicular are similar to the whole triangle and to each other. 6

**32.** The expenditure for the consumption of water per month by 100 families is given below: 6

| Expenditure (in Rs) | No. of Families |
|---|---|
| 30 – 40 | 12 |
| 40 – 50 | 18 |
| 50 – 60 | 20 |
| 60 – 70 | 15 |
| 70 – 80 | 12 |
| 80 – 90 | 11 |
| 90 – 100 | 6 |
| 100 – 110 | 4 |
| 110 – 120 | 2 |

Find the mean monthly expenditure of the families on water. Find also the quartiles of the expenditure.

## 2014

**1.** If $|x+5| = -x$, then the value of $x$ is 1

**(A)** 0. **(B)** 5. **(C)** $-\frac{5}{2}$. **(D)** $\frac{5}{2}$.

**2.** The pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has a unique solution if 1

**(A)** $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. **(B)** $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

**(C)** $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. **(D)** $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

**3.** The 15$^{\text{th}}$ term of an AP exceeds the 22$^{\text{nd}}$ term by 35. The common difference of the AP is 1

**(A)** $-5$. **(B)** 5. **(C)** $-7$. **(D)** 7.

**4.** The circumference of a circle is 44 cm. Its area in square centimeters is

**(A)** 38. **(B)** 38.5. **(C)** 40. **(D)** 40.5. 1

**5.** If $A$ and $B$ are two independent events of a random experiment, then $P(AB) =$ 1

**(A)** $P(A) + P(B)$. **(B)** $P(A) - P(B)$.

**(C)** $P(A) \cdot P(B)$. **(D)** $\frac{P(A)}{P(B)}$.

**6.** Write the statement of "Euclid's division lemma". 1

**7.** Find the constant remainder when $2x^2 - 3x + 2$ is divided by $x - 1$. 1

**8.** Define an arithmetic progression. 1

**9.** The $n^{\text{th}}$ term of a sequence is $n^2 + 3$. Is the sequence an AP? 1

**10.** The length of the sides of a triangle are 7 cm, 24 cm and 25 cm. Determine if the triangle is a right triangle or not. 1

**11.** Write a Pythagorean relation between the trigonometric ratios $\sec A$ and $\tan A$. 1

**12.** Write the formula for the volume of a frustum of cone. 1

**13.** When are events of a random experiment said to be equally likely? 1

**14.** If $x, y, z \in \mathbb{R}$ and $x + y = x + z$, then prove that $y = z$. 2

**15.** Find the quadratic equation whose roots are $\alpha$ and $\beta$. 2

**16.** Find the sum of the first 25 terms of the AP: $20, 17, 14, 11, \ldots$. 2

**17.** If $\cos A = \dfrac{4}{5}$, find $\sin A$ and $\tan A$. 2

**18.** A die is thrown once. Find the probability of getting a number greater than 4. 2

**19.** Factorise $a^2(b + c) + b^2(a + b) + c^2(a + b) + 2abc$. 3

**20.** Using the prime factorisation method, find the HCF and the LCM of 420, 504 and 924. 3

**21.** Solve graphically: 3

$$4x + 3y = 12 \text{ and } 5x - 3y = 15.$$

**22.** Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. 3

**23.** Find the values of trigonometric ratios of $45°$. 3

**24.** A chord of a circle of radius 12 cm subtends an angle of $60°$ at the centre. Find the area of the corresponding minor segment. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$.) 3

**25.** State and prove factor theorem. 4

**26.** A motor boat whose speed is 18 km/hr in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. 4

*Or*

A number consists of two digits. The product of the digits is 20. When 9 is added to the number, the digits interchange their places. Find the number. 4

**27.** In what ratio is the line segment joining the points $(-2, 3)$ and $(3, 8)$ divided by the $y$-axis? Also, find the coordinates of the point of division. 4

**28.** Construct a pair of tangents to a circle from an external point. Write the steps of construction. $2+3$

**29.** The angle of elevation of a bird from the eye of a man on the bank of a pond is $30°$ and the angle of depression of its reflection in the pond is $60°$. Find the height of the bird above the pond if the height of the eye above water surface is 1.5 metres. 5

*Or*

A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height $h$. At a point on the plane, the angle of elevation of the bottom of the flagstaff is $\alpha$ and that of the top of the flagstaff is $\beta$. Prove that the height of the tower is

$$\frac{h\tan\alpha}{\tan\beta - \tan\alpha}.$$ 5

**30.** A metallic sphere of radius 6 cm is melted and recast to form a cylinder of height 32 cm. Find the total surface area of the cylinder. (Take $\pi = 3.14$.) 5

**31.** State and prove SSS similarity theorem. 6

*Or*

Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the other two sides. 6

**32.** In the following distribution, the frequencies of two classes are missing. However, the mean of the data is given to be 50. Find the missing frequencies. 6

| Class | Frequency |
|---|---|
| 0 – 20 | 17 |
| 20 – 40 | ⋯ |
| 40 – 60 | 32 |
| 60 – 80 | ⋯ |
| 80 – 100 | 19 |
| Total | 120 |

# Answers, Hints, Solutions

## 2010

**1.** (D) $p(a)$. (Remainder theorem.)

**2.** (B) 1. $(\cot 55^\circ = \cot(90^\circ - 35^\circ) = \tan 35^\circ.)$

**3.** (A) $20\sqrt{3}$. $\left(\frac{\text{height of the tower}}{\text{length of the shadow}} = \tan 60^\circ = \sqrt{3}.\right)$

**4.** (C) $-4$. $(\because\ a(3-2) + 2(2-0) + 0(0-3) = 0.)$

**5.** (A) $\dfrac{\theta\pi r^2}{360}$.

**6.** The actual division of $x^3 - 1$ by $x^2 + x + 1$ is as follows:

$$\begin{array}{r|l} & \quad\quad\quad\quad\quad\quad\; x - 1 \\ \hline x^2 + x + 1 & x^3 + 0x^2 + 0x - 1 \\ & \underline{x^3 + \;\; x^2 + \;\; x} \\ & \quad\quad -x^2 - \;\; x - 1 \\ & \quad\quad \underline{-x^2 - \;\; x - 1} \\ & \quad\quad\quad\quad\quad\quad\quad\; 0 \end{array}$$

Hence, the required quotient is $x - 1$.

**7.** A pair of linear equations is said to be a dependent pair if one equation is obtained from the other on multiplying by a constant.

**8.** The given pair of equations has a unique solution if $\dfrac{2}{4} \neq \dfrac{3}{k}$, i.e., if $k \neq 6$.

**9.** The quantity $b^2 - 4ac$ is called the discriminant of the quadratic equation $ax^2 + bx + c = 0$.

**10.** A sequence $a_1, a_2, \ldots, a_n, \ldots$ is called an arithmetic progression (AP) if the difference $(a_{n+1} - a_n)$ is constant for all $n \in \mathbb{N}$.

**11.** **Pythagoras theorem:** In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

**12.** The coordinates of the mid-point of the line segment joining $(x_1, y_1)$ and $(x_2, y_2)$ are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

**13.** Here, $r = 3$ cm, $h = 7$ cm. So, the curved surface area of the cylinder $= 2\pi rh = 2 \times \dfrac{22}{7} \times 3 \times 7 = 132\ \text{cm}^2$.

**14.** **(i) Sample space:** The totality of all the possible outcomes of an experiment is called the sample space of the experiment.

**(ii) Event:** Any component of a sample space is an event.

**15.** Clearly, the equality holds for $x = 0$. If $x \neq 0$, then

$$\begin{aligned}|-x| &= \text{ the greater of } -x \text{ and } -(-x)\\ &= \text{ the greater of } -x \text{ and } x\\ &= \text{ the greater of } x \text{ and } -x\\ &= |x|.\end{aligned}$$

Thus, $|-x| = |x|$ for any $x \in \mathbb{R}$. □

**16.** We proceed as follows:

$$\begin{aligned}&ab(a+b) + bc(b+c) + ca(c+a) + 3abc\\ &\qquad = \{ab(a+b) + abc\} + \{bc(b+c) + abc\} + \{ca(c+a) + abc\}\\ &\qquad = ab(a+b+c) + bc(b+c+a) + ca(c+a+b)\\ &\qquad = (a+b+c)(bc+ca+ab).\end{aligned}$$

**17.** Let $a$ be the first term and $d$ the common difference of an AP. Also, let $S_n$ denote the sum of the first $n$ terms of the $AP$. Now, we have

$$S_n = a + (a+d) + \cdots + [a + (n-1)d]. \tag{1}$$

Rewriting the terms in reverse order, we get

$$S_n = [a + (n-1)d] + \cdots + (a+d) + a. \tag{2}$$

Adding (1) and (2) termwise, we get

$$\begin{aligned}2S_n &= \underbrace{[2a + (n-1)d] + \cdots + [2a + (n-1)d]}_{n \text{ terms}}\\ \Longrightarrow 2S_n &= n[2a + (n-1)d]\\ \Longrightarrow S_n &= \frac{n}{2}[2a + (n-1)d].\end{aligned}$$

□

**18.** Here, $a_{n+1} = 3(n+1) + 2 = 3n + 5$. So, $a_{n+1} - a_n = 3$, which is a constant for all $n \in \mathbb{N}$. Hence, the sequence is an AP.

**19.** We know that the tangent to the circle is perpendicular to the radius of the circle at the point of contact. Applying Pythagoras theorem, the radius $= \sqrt{5^2 - 4^2} = 3$ cm.

**20.** **Factor theorem:** If $p(x)$ is a polynomial of degree $\geq 1$ and $a$ is any real number, then $x - a$ is a factor of $p(x)$ if and only if $p(a) = 0$.

**Proof:** Let us suppose that $p(a) = 0$.
By remainder theorem, we know that $p(a)$ is the remainder when $p(x)$ is divided by $x - a$.

$$\begin{aligned} \therefore\ p(x) &= (x-a)q(x) + p(a) \text{ for some polynomial } q(x) \\ &= (x-a)q(x) \quad (\because p(a) = 0). \end{aligned}$$

Hence, $x - a$ is a factor of $p(x)$.

Conversely, let us suppose that $x - a$ is a factor of $p(x)$.
Then $p(x) = (x - a)q(x)$ for some polynomial $q(x)$.
Putting $x = a$, we get

$$p(a) = (a - a)q(a) = 0 \cdot q(a) = 0. \qquad \square$$

**21.** $x^6 + 8x^3 + 27 = (x^2)^3 + (-x)^3 + 3^3 - 3(x^2)(-x)(3)$. Take $a = x^2$, $b = -x$, $c = 3$. We know that

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca).$$

Hence, $x^6 + 8x^3 + 27 = (x^2 - x + 3)(x^4 + x^3 - 2x^2 + 3x + 9)$.

**22.** The given pair of equations is

$$3x + y = 9, \qquad (1)$$
$$2x - 3y + 16 = 0. \qquad (2)$$

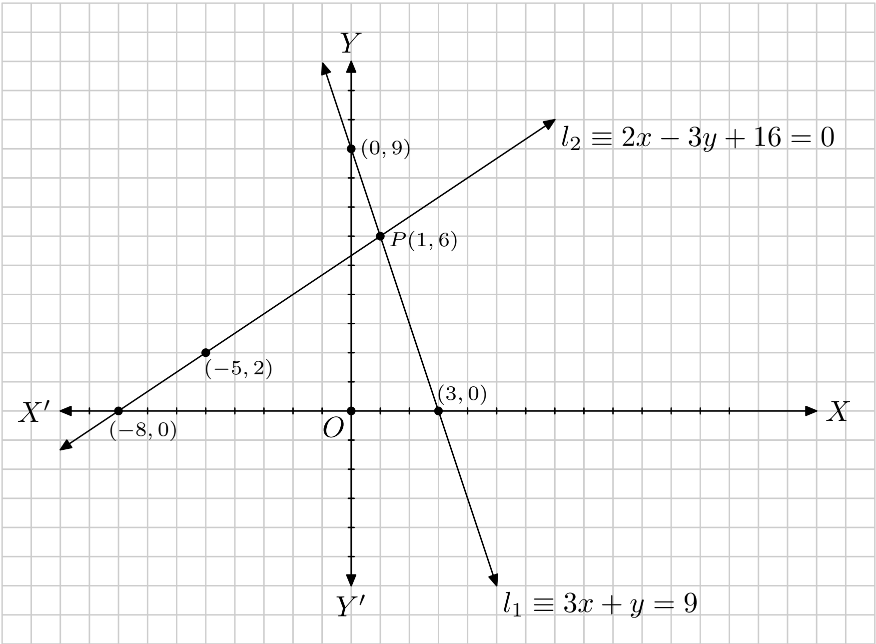
Some points satisfying equations (1) and (2) are respectively given below:

| $x$ | 3 | 1 | 0 |
|---|---|---|---|
| $y$ | 0 | 6 | 9 |

and

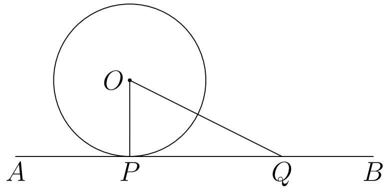
| $x$ | $-8$ | $-5$ | 1 |
|---|---|---|---|
| $y$ | 0 | 2 | 6 |

The points in the two tables are plotted and joined together in the Cartesian plane (as shown in figure) to get the straight lines $l_1$ and $l_2$ respectively. Thus, $l_1$ is the graph of $3x + y = 9$ and $l_2$ is the graph of $2x - 3y + 16 = 0$.




It is seen from the figure that the two straight lines intersect at the point $P(1, 6)$. Therefore, the coordinates of $P$ will satisfy both the equations (1) and (2). Hence, $x = 1$ and $y = 6$.

**23.** **Given:** $AB$ is a tangent to a circle with centre $O$ and $P$ is the point of contact.



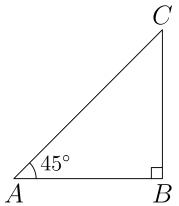

**To prove:** $OP \perp AB$.

**Construction:** Let us take a point $Q$ on $AB$ other than $P$. Join $OQ$.

**Proof:** Since $Q$ is not the point of contact, $Q$ lies outside the circle and so $OQ > OP$. Since this is true for all the points other than $P$, $OP$ is the shortest of all segments drawn from the centre $O$ to the points on the tangent $AB$. But the shortest segment that can be drawn from a point to a line is the perpendicular segment from the point to the line. Hence, $OP \perp AB$. □

**24.** **Trigonometric ratios of** $45°$**:** Let us consider a right triangle $ABC$, right angled at $B$ and $\angle A = 45°$. Clearly, $\angle C = 45°$ and so $AB = BC$.Suppose $AB = BC = k$ for some positive number $k$. Then, by Pythagoras theorem, we have $AC^2 = AB^2 + BC^2 = k^2 + k^2 = 2k^2$. So, $AC = \sqrt{2}k$. Thus, by definitions of the trigonometric ratios, we have

$$\sin 45^\circ = \frac{BC}{AC} = \frac{k}{\sqrt{2}k} = \frac{1}{\sqrt{2}},$$
$$\operatorname{cosec} 45^\circ = \frac{AC}{BC} = \frac{\sqrt{2}k}{k} = \sqrt{2},$$
$$\cos 45^\circ = \frac{AB}{AC} = \frac{k}{\sqrt{2}k} = \frac{1}{\sqrt{2}},$$
$$\sec 45^\circ = \frac{AC}{AB} = \frac{\sqrt{2}k}{k} = \sqrt{2},$$
$$\tan 45^\circ = \frac{BC}{AB} = \frac{k}{k} = 1,$$
$$\cot 45^\circ = \frac{AB}{BC} = \frac{k}{k} = 1.$$




**25.** We heve

$$\frac{\sin\theta}{\sec\theta+1} + \frac{\sin\theta}{\sec\theta-1} = \frac{2\sin\theta\sec\theta}{\sec^2\theta-1} = \frac{2\tan\theta}{\tan^2\theta} = 2\cot\theta. \qquad \square$$

**26.** $\operatorname{lcm}[6, 9, 15] = 90$. So, the required number is of the form $90k+a$, where $k, a \in \mathbb{Z}$ such that $0 < k$, $0 \le a < 6$. Now, $90k+a = (17\times5+5)k+a = 17\times5k+5k+a$. So, $90k+a$ is multiple of 17 if $5k+a$ is a multiple of 17. Such least multiple is obtained when $5k+a = 17$. So, $k = 3$ and $a = 2$. Hence, the required number is $90\times3+2 = 272$.

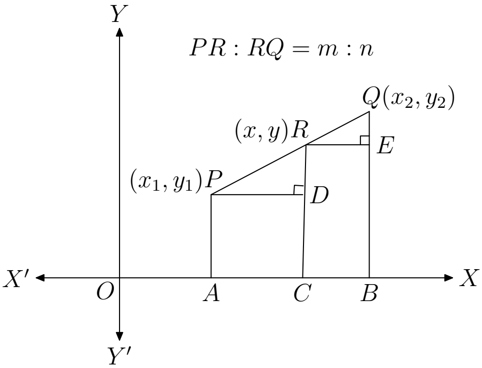
**27. Solution of a quadratic equation by completing perfect square:** Consider the quadratic equation $ax^2+bx+c = 0$, $a \neq 0$. Dividing throughout by $a$, we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$\implies x^2 + 2\times x\times\frac{b}{2a} + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$$
$$\implies \left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$
$$\implies x + \frac{b}{2a} = \frac{\pm\sqrt{b^2-4ac}}{2a}$$
$$\therefore x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

**28. 1st problem: Proof:**

Let $R(x, y)$ be the point which divides the line segment joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $m : n$.

Construction: Draw $PA$, $QB$, $RC$ perpendicular to $x$-axis, and draw $PD \perp RC$ and $RE \perp QB$.




Now, we have

$$\left.\begin{aligned} PD &= AC = OC - OA = x - x_1, \\ RD &= RC - DC = RC - PA = y - y_1, \\ RE &= CB = OB - OC = x_2 - x, \\ QE &= QB - EB = QB - RC = y_2 - y. \end{aligned}\right\} \quad (1)$$

In $\triangle$s $PDR$ and $REQ$,

$$\begin{aligned} &\angle PDR = \angle REQ = 90^\circ, \\ &\angle RPD = \angle QRE, \text{ being corresponding angles.} \\ \therefore\ &\triangle PDR \sim \triangle REQ \quad \text{(by AA similarity).} \\ \Longrightarrow\ &\frac{PR}{RQ} = \frac{PD}{RE} = \frac{RD}{QE}. \end{aligned} \quad (2)$$

Using (1) in (2), we have

$$\frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}. \quad (3)$$

From the first two ratios of (3), we get

$$mx_2 - mx = nx - nx_1 \implies x = \frac{mx_2 + nx_1}{m + n}.$$

From the first and the last ratios of (3), we get

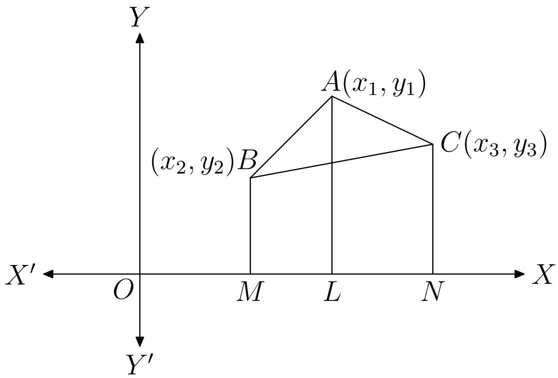
$$my_2 - my = ny - ny_1 \implies y = \frac{my_2 + ny_1}{m + n}.$$

Hence, the coordinates of the point $R$ which divides the line segment joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $m : n$ are

$$\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right). \qquad \square$$

*Or*

**2nd problem: Proof:** Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of $\triangle ABC$ (see figure below). Draw $AL$, $BM$ and $CN$ perpendicular to $x$-axis.




From the figure, we have

$$\text{ar}(\triangle ABC) = \text{ar}(\text{trapezium } ABML) + \text{ar}(\text{trapezium } ALNC) - \text{ar}(\text{trapezium } BMNC).$$

We know that the area of a trapezium

$$= \frac{1}{2} \times (\text{sum of the parallel sides}) \times (\text{distance between them}).$$

So, we have

$$\begin{aligned}\text{ar}(\triangle ABC) &= \frac{1}{2}(AL + BM)ML + \frac{1}{2}(AL + CN)LN \\ &\quad - \frac{1}{2}(BM + CN)MN \\ &= \frac{1}{2}(y_1 + y_2)(x_1 - x_2) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1) \\ &\quad - \frac{1}{2}(y_2 + y_3)(x_3 - x_2) \\ &= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)].\end{aligned}$$

Note that if the three points are taken in anticlockwise sense, $\text{ar}(\triangle ABC)$ will be positive, while if the points are taken in clockwise sense, then $\text{ar}(\triangle ABC)$ will be negative. In order to avoid the negativity of area, the absolute value of the area is taken and hence

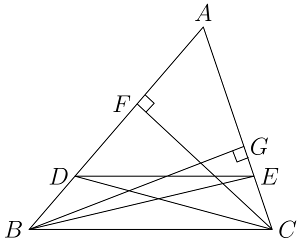
$$\text{ar}(\triangle ABC) = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|. \qquad \square$$

**29.** Here, $h = 27$ cm, $r = 16$ cm, the volume of cone $= \frac{1}{3}\pi r^2 h = 16^2 \times 9\pi$. If $R$ is the radius of the sphere, we have

$$16^2 \times 9\pi = \frac{4}{3}\pi R^3 \implies R^3 = 4^3 \times 3^3 \implies R = 12 \text{ cm}.$$

Hence, the curved surface area of the sphere $= 4\pi R^2 = 1810.29$ cm$^2$.

**30. 1st problem: Basic proportionality theorem:** If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the other two sides in the same ratio.




**Given:** $ABC$ is a triangle. $DE$ is a line drawn parallel to $BC$ intersecting $AB$ at $D$ and $AC$ at $E$.

**To prove:** $\frac{AD}{DB} = \frac{AE}{EC}$.

**Construction:** Join $BE$ and $CD$. Draw $CF \perp AB$ and $BG \perp AC$.

**Proof:** Since triangles standing on the same base and between the same parallel lines have the same area,

$$\text{ar}(\triangle DBC) = \text{ar}(\triangle EBC). \tag{1}$$

Therefore, we have

$$\begin{aligned} &\text{ar}(\triangle ABC) - \text{ar}(\triangle DBC) = \text{ar}(\triangle ABC) = \text{ar}(\triangle EBC) \\ \implies &\text{ar}(\triangle ADC) = \text{ar}(\triangle AEB). \end{aligned} \tag{2}$$

Dividing (1) by (2), we have

$$\begin{aligned} &\frac{\text{ar}(\triangle ADC)}{\text{ar}(\triangle DBC)} = \frac{\text{ar}(\triangle AEB)}{\text{ar}(\triangle EBC)} \\ \implies &\frac{\frac{1}{2} \times AD \times CF}{\frac{1}{2} \times DB \times CF} = \frac{\frac{1}{2} \times AE \times BG}{\frac{1}{2} \times EC \times BG} \\ \implies &\frac{AD}{DB} = \frac{AE}{EC}. \end{aligned}$$

□

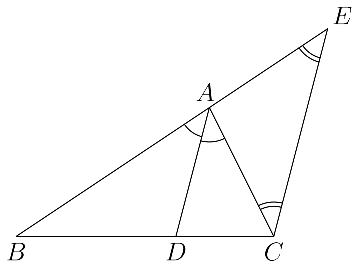
*Or*

**2nd problem:**

**Given:** $ABC$ is a triangle in which $AD$ is the internal bisector of $\angle A$ meeting $BC$ at $D$.

**To prove:** $\dfrac{BD}{DC} = \dfrac{AB}{AC}$.

**Construction:** Draw $CE$ parallel to $DA$ meeting $BA$ produced at $E$.




**Proof:** By construction $DA \| CE$. So, by basic proportionality theorem, we have

$$\frac{BD}{DC} = \frac{AB}{AE}. \tag{1}$$

Now, $DA \| CE$ and $AC$, $BE$ are transversal lines. So, we have

$$\angle BAD = \angle AEC \quad \text{and} \quad \angle DAC = \angle ACE. \tag{2}$$

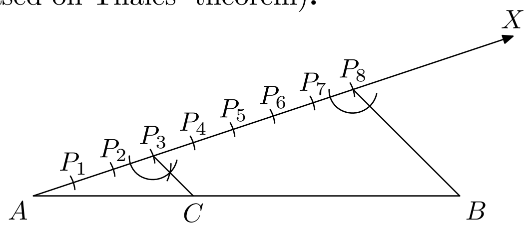
But $AD$ bisects $\angle BAC$. Therefore, $\angle BAD = \angle DAC$. By (2), we have

$$\angle AEC = \angle ACE \implies AC = AE.$$

Putting this value in (1), we have

$$\frac{BD}{DC} = \frac{AB}{AC}. \qquad \square$$

**31. Diagram** (based on Thales' theorem)**:**




**Steps of construction:**

**(1)** Draw a line segment $AB$ of any length.

**(2)** Draw a ray $AX$ inclined to $AB$ at an acute angle.

**(3)** Mark eight $(= 3+5)$ points $P_1, P_2, \ldots, P_8$ on $AX$ such that $AP_1 = P_iP_{i+1}$ for all $i = 1, 2, \ldots, 7$.

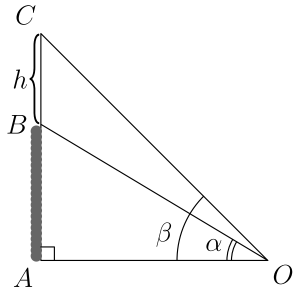
**(4)** Join $P_8B$ and through the point $P_3$ draw a line parallel to $P_8B$ meeting $AB$ at $C$.

Thus, the point $C$ on $AB$ divides the line segment $AB$ in the ratio $3 : 5$.

**32. 1st problem:** Let $AB$ be the tower and $BC$ be the flag. Let $O$ be the point on the horizontal plane which makes angles of elevation $\alpha$ and $\beta$ respectively at $B$ and $C$. In right $\triangle$s $ABO$ and $ACO$, we have

$$\frac{AB}{AO} = \tan\alpha \qquad (1)$$

$$\text{and } \frac{AC}{AO} = \tan\beta. \qquad (2)$$




Dividing (1) by (2), we get

$$\frac{AB}{AC} = \frac{\tan\alpha}{\tan\beta}$$

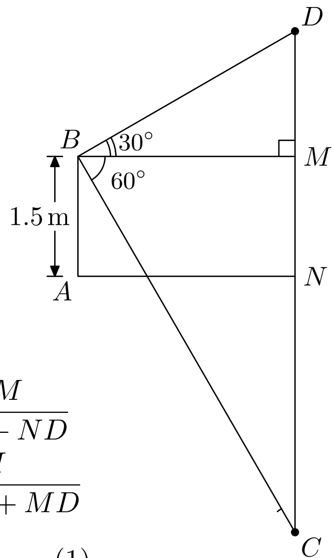
$$\implies AB\tan\beta = AC\tan\alpha = (AB+h)\tan\alpha$$

$$\implies AB(\tan\beta - \tan\alpha) = h\tan\alpha$$

$$\implies AB = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}. \qquad \square$$

*Or*

**2nd problem:** Let $B$ be the eye of the man, $AN$ be the water level, $C$ be the reflection of the bird on the pond and $ND$ be the height of the bird at $D$ above the pond as shown in the figure.




Since the distance between a mirror and an object is the same as the distance between the mirror and the image of the object, $ND = NC$. Now, in right $\triangle BMC$,

$$\cot 60^\circ = \frac{BM}{MC} = \frac{BM}{MN+NC} = \frac{BM}{MN+ND}$$

$$\implies \frac{1}{\sqrt{3}} = \frac{BM}{MN+NM+MD} = \frac{BM}{1.5+1.5+MD}$$

$$\implies BM = \frac{3+MD}{\sqrt{3}} \text{ m.} \qquad (1)$$

Using (1) in right $\triangle BMD$, we have

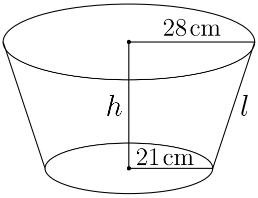
$$\cot 30^\circ = \frac{BM}{MD} \implies \sqrt{3} = \frac{3+MD}{\sqrt{3}MD} \implies MD = 1.5 \text{ m.}$$

Now, $DK = DM + MK = 1.5 + 1.5 = 3$.

Hence, the height of the bird above the pond is 3 m.

**33.** On rolling two fair dice, $6 \times 6 = 36$ outcomes are possible. Out of these 36 outcomes, there are 6 outcomes in which the sum of the points is 7, namely (1,6), (6,1), (2,5), (5,2), (3,4) and (4,3), and there are 5 outcomes in which the sum of the points is 8, namely (2,6), (6,2), (3,5), (5,3) and (4,4). So, $P(\text{sum is 7}) = \frac{6}{36} > \frac{5}{36} = P(\text{sum is 8})$. It is more likely to happen that the sum is 7 than that the sum is 8.

**34.** Here, $r_1 = 28$ cm, $r_2 = 21$ cm. Let $h$ be the height of the bucket. Since the volume of the bucket is 45584 cm$^3$, we have




$$\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2) = 45584$$
$$\Longrightarrow \frac{22}{7}h(28^2 + 21^2 + 28 \times 21) = 3 \times 45584$$
$$\Longrightarrow h = 24 \text{ cm}.$$

Therefore, the slant height $l$ of the bucket is given by

$$l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{24^2 + (28 - 21)^2} = 25 \text{ cm}.$$

Hence, the surface area of the bucket

$$= \pi l(r_1 + r_2) + \pi r_2^2$$
$$= \frac{22}{7} \times 25 \times (28 + 21) + \frac{22}{7} \times 21^2$$
$$= 3850 + 1386$$
$$= 5236 \text{ cm}^2.$$

**35.** Taking $a = 55$, the given table is modified as follows:

| Age group | $x_i$ | $u_i = \frac{x_i - 55}{10}$ | $f_i$ | c.f. | $f_i u_i$ |
|---|---|---|---|---|---|
| 0 – 10 | 5 | -5 | 51 | 51 | -255 |
| 10 – 20 | 15 | -4 | 55 | 106 | -220 |
| 20 – 30 | 25 | -3 | 78 | 184 | -234 |
| 30 – 40 | 35 | -2 | 75 | 259 | -150 |
| 40 – 50 | 45 | -1 | 62 | 321 | -62 |
| 50 – 60 | 55 | 0 | 47 | 368 | 0 |
| 60 – 70 | 65 | 1 | 23 | 391 | 23 |
| 70 – 80 | 75 | 2 | 7 | 398 | 14 |
| 80 – 90 | 85 | 3 | 2 | 400 | 6 |
| 90 – 100 | 95 | 4 | 0 | 400 | 0 |
| | | | $N = 400$ | | $\sum f_i u_i = -878$ |

Here, $h = 10$, $N = 400$ and $\overline{u} = \dfrac{\sum f_i u_i}{N} = \dfrac{-878}{400} = -2.195$.
Hence, $\overline{x} = a + h\overline{u} = 33.05$.
Again, $\dfrac{N}{2} = 200$. The cumulative frequency just greater than 200 is 259. So, the median class is $30 - 40$. Now, $l = 30$, $f = 75$, $c = 184$.

$$\therefore \text{ median} = l + \frac{\frac{N}{2} - c}{f} \times h = 30 + \frac{\frac{400}{2} - 184}{75} \times 10 = 32.13.$$

## 2011

**1.** (C) $-4$. (Factor theorem.)

**2.** (A) $(a+b)(b+c)(c+a)$.

**3.** (D) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$.

**4.** (B) 9. (Discriminant must be zero.)

**5.** (C) $\pi r^2$.

**6.** An algebraic expression is said to have cyclic factors if it has as its factors all the expressions obtained by cyclical replacement in any one of the factors.

**7.** A real number $\alpha$ is called a root of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, if $a\alpha^2 + b\alpha + c = 0$.

**8.** Here, $a = 1$, $d = 2$, $n = 10$. So, $S_{10} = \dfrac{10}{2}\,[2 \times 1 + (10 - 1) \times 2] = 100$.

**9.** A tangent to a circle is a line that intersects the circle at exactly one point.

**10.** $\angle APB = 180^\circ - 130^\circ = 50^\circ$ is the required angle.

**11.** The coordinates of the mid-point are $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

**12.** $\tan 40^\circ \tan 45^\circ \tan 50^\circ = \tan 40^\circ \times 1 \times \cot(90^\circ - 50^\circ) = 1$.

**13.** The arc length $= \dfrac{\theta}{360} \times 2\pi r = \dfrac{\pi r \theta}{180}$.

**14.** We know that $|ab| = |a||b|$. So, $|x|^2 = |x||x| = |xx| = |x^2| = x^2$.

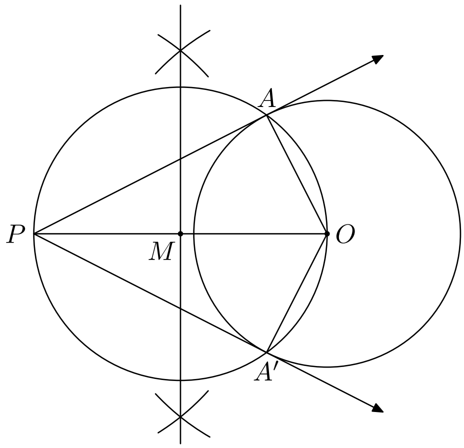
**15.** Let $a$ be the first term and $d$ the common difference of an AP. Then the corresponding AP is

$$\begin{aligned} a_1 &= a = a + (1-1)d, \\ a_2 &= a + d = a + (2-1)d, \\ a_3 &= a + 2d = a + (3-1)d, \\ &\vdots \end{aligned}$$

Looking at the pattern, we notice that in $n^{\text{th}}$ term, the coefficient of $d$ is always less by 1 than $n$. Hence, the $n^{\text{th}}$ term of the AP is $a_n = a + (n-1)d$. □

**16.** Here, the sum of the roots $= (3+\sqrt{5}) + (3-\sqrt{5}) = 6$ and the product of the roots $= (3+\sqrt{5}) \times (3-\sqrt{5}) = 9 - 5 = 4$. Hence, the required quadratic equation is $x^2 - 6x + 4 = 0$.

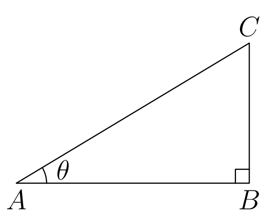
**17.** We construct a pair of tangents to a circle from an external point $P$ as follows:




The lines $PA$ and $PA'$ are the required tangents.

**18.** $ABC$ be a right triangle, right angled at $B$. By Pythagoras theorem, we have $AB^2 + BC^2 = AC^2$.

Dividing throughout by $AC$, we have




$$\begin{aligned} &\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2} \\ \implies &\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = 1 \\ \implies &(\cos A)^2 + (\sin A)^2 = 1. \end{aligned}$$

□

**19.** We know that any integer $a$ is of the form $4q+r$, $0 \leq r < 4$. If $r = 0,\ 2$, then $a$ is even. For odd $a$, we have $r = 1,\ 3$. Thus, $a$ is of the form $4k+1$ or $a$ is of the form $4q+3 = 4(q+1)-1 = 4k-1$, where $k = q+1$.

**20.** **Remainder theorem:** Let $p(x)$ be any polynomial of degree greater than or equal to one and let $a$ be any real number. If $p(x)$ is divided by the linear polynomial $x - a$, then the remainder is $p(a)$.

**Proof:** Suppose that when $p(x)$ is divided by $x - a$, the quotient is $q(x)$ and the (constant) remainder is $R$. Then

$$p(x) = (x - a) \cdot q(x) + R.$$

Since this relation is true for all values of $x$, putting $x = a$, we have

$$p(a) = (a - a) \cdot q(a) + R = 0 \cdot q(a) + R = R.$$

This shows that the remainder is $p(a)$. □

**21.** Since $\alpha$ and $\beta$ are the roots of $ax^2 + bx + c = 0$, we have

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Now, $\alpha\beta$ is given by

$$\begin{aligned} \alpha\beta &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{4a^2} \\ &= \frac{b^2 - (b^2 - 4ac)}{4a^2} \\ &= \frac{4ac}{4a^2} \\ &= \frac{c}{a}. \end{aligned}$$ □

**22.** Here, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{a}{b}$. So, $\sin\theta = ak$ and $\cos\theta = bk$ for some positive number $k$. Now, we have

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a(ak) - b(bk)}{a(ak) + b(bk)} = \frac{\left(a^2 - b^2\right)k}{\left(a^2 + b^2\right)k} = \frac{a^2 - b^2}{a^2 + b^2}.$$ □

**23.** See 2nd problem, Q.**28.**, 2010 on page 24.

**24.** Volume of the cylinder $= \pi \times 3^2 \times 24 = 216\pi$ cm$^3$. If $h$ is the height of the cone, then $\frac{1}{3}\pi \times 6^2 \times h = 216\pi$. So, $h = 18$ cm.

**25.** **Euclid's algorithm** for finding the HCF of two given positive integers:

**Step 1.** Find the quotient and remainder of the division of the greater number by the smaller.

**Step 2.** If the remainder is zero, then the divisor is the HCF.

**Step 3.** Else, taking the previous remainder as the new divisor and the previous divisor as the new dividend, find the quotient and the remainder.

**Step 4.** Continue the process till the remainder is zero. The last divisor is the required HCF.

**26.** Let Rs $x$ be the sum invested at 12% and Rs $y$ be the sum invested at 15%. By the given conditions, we get

$$x + y = 36000 \tag{1}$$

$$\text{and } \frac{12}{100}x + \frac{15}{100}y = 4890 \implies 4x + 5y = 163000. \tag{2}$$

Multiplying (1) by 4 and subtracting from (2), we get

$$5y - 4y = 163000 - 36000 \times 4 \implies y = 19000.$$

Using this value of $y$ in (1), we get

$$x + 19000 = 36000 \implies x = 17000.$$

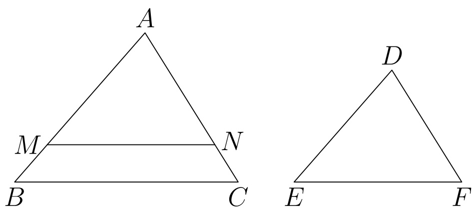
Hence, the man invested Rs 17000 at 12% and Rs 19000 at 15%.

**27.** When two dice are thrown, there are $6 \times 6$ possible outcomes. Out of these 36 exhaustive cases, the sum is 10 in three cases: $(4, 6)$, $(5, 5)$, $(6, 4)$. Thus, the required probability $= \dfrac{3}{36} = \dfrac{1}{12}$.

**28.** **1st problem: SAS similarity:** If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the triangles are similar.

**Given:** Two $\triangle$s $ABC$ and $DEF$ in which $\angle A = \angle D$ and $\dfrac{AB}{DE} = \dfrac{AC}{DF}$.

**To prove:** $\triangle ABC \sim \triangle DEF$.




**Proof:** If $AB = DE$, then

$$\frac{AC}{DF} = \frac{AB}{DE} = 1$$
$$\implies AC = DF.$$

By SAS congruence, $\triangle ABC \cong \triangle DEF$. Hence, $\triangle ABC \sim \triangle DEF$.

Now, if $AB \neq DE$, then one of them is greater than the other. Without loss of generality, we can assume that $AB > DE$. Take points $M$ and $N$ on $AB$ and $AC$ respectively such that $AM = DE$ and $AN = DF$. Join $MN$. See the figure above. In $\triangle$s $AMN$ and $DEF$,

$$AM = DE, \quad AN = DF \quad \text{and} \quad \angle A = \angle D.$$

By SAS congruence, $\triangle AMN \cong \triangle DEF$. So, we have

$$\angle AMN = \angle DEF, \quad \angle ANM = \angle DFE. \tag{1}$$

Now, it is given that

$$\frac{AB}{DE} = \frac{AC}{DF} \implies \frac{AB}{AM} = \frac{AC}{AN} \implies \frac{AM}{AB} = \frac{AN}{AC}.$$

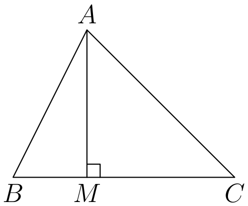
By the converse of basic proportionality theorem, $MN \| BC$. So,

$$\angle ABC = \angle AMN = \angle DEF, \quad \angle ACB = \angle ANM = \angle DFE \quad \text{(by (1))}.$$

Hence, by AA similarity, $\triangle ABC \sim \triangle DEF$. $\square$

*Or*

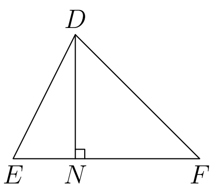
**2nd problem: Given:** $ABC$ and $DEF$ are two similar triangles.




**To prove:** $\dfrac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \dfrac{AB^2}{DE^2} = \dfrac{BC^2}{EF^2} = \dfrac{AC^2}{DF^2}$.

**Construction:** Draw $AM \perp BC$, $DN \perp EF$.

**Proof:** Since $\text{ar}(\triangle ABC) = \dfrac{1}{2} \times BC \times AM$ and $\text{ar}(\triangle DEF) = \dfrac{1}{2} \times EF \times DN$,




$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{BC \times AM}{EF \times DN}. \tag{1}$$

Now, $\triangle ABC \sim \triangle DEF$. So, we have

$$\frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF}. \tag{2}$$

In $\triangle$s $ABM$ and $DEN$, we have

$$\angle ABM = \angle DEN \quad (\because\ \triangle ABC \sim \triangle DEF),$$

$$\angle AMB = \angle DNE = 90^\circ.$$

Therefore, by AA similarity, $\triangle ABM \sim \triangle DEN$. So, we have

$$\frac{AM}{DN} = \frac{AB}{DE} \implies \frac{AM}{DN} = \frac{BC}{EF} \quad \text{(by (2))}. \tag{3}$$

By (1) and (3), we get

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{BC^2}{EF^2}.$$

Hence, by (1), we have

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}. \qquad \square$$

**29.** **1st problem:** Let $AB$ be the tower. Let $C$ and $D$ be the first and the second points respectively.

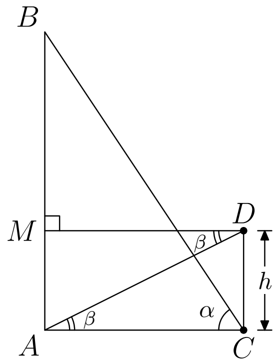
In right $\triangle$s $ABC$ and $ADC$,

$$\frac{AB}{AC} = \tan\alpha \tag{1}$$

$$\text{and } \frac{DC}{AC} = \tan\beta$$

$$\implies \frac{AM}{AC} = \tan\beta$$

$$\implies AC = \frac{h}{\tan\beta}. \tag{2}$$



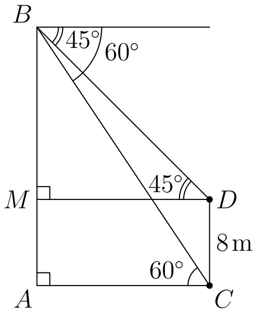

Multiplying (1) and (2), we get

$$AB = \tan\alpha\left(\frac{h}{\tan\beta}\right) = h\tan\alpha\cot\beta. \qquad \square$$

*Or*

**2nd problem:** Let $AB$ represent the tower and $CD$ represent the 8 m tall tree.
In right $\triangle BMD$, we have

$$\frac{BM}{MD} = \tan 45^\circ = 1 \implies BM = MD = AC. \tag{1}$$




Again, in right $\triangle BAC$, we have

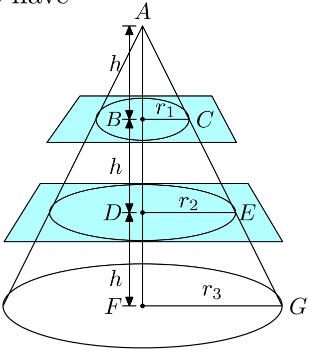
$$\frac{AB}{AC} = \tan 60^\circ = \sqrt{3}$$

$$\implies AB = AC\sqrt{3} = BM\sqrt{3} \quad \text{(by (1))}$$
$$\implies AB = (AB-8)\sqrt{3} = AB\sqrt{3} - 8\sqrt{3}$$
$$\implies AB(\sqrt{3}-1) = 8\sqrt{3}$$
$$\implies AB = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 12 + 4\sqrt{3}.$$

Hence, the height of the tower is $12 + 4\sqrt{3}$ m.

**30.** **1st problem:** Let $r_1$, $r_2$ and $r_3$ ($r_1 < r_2 < r_3$) be the radii of the circular bases at the cut points and $h$ be the height of each part as shown in the adjoining figure.

Let $V_1$ be the volume of the cone with radius $r_1$, $V_2$ be the volume of the frustum with bases radii $r_1$ and $r_2$ and $V_3$ be the volume of the frustum with bases radii $r_2$ and $r_3$. Then we have




$$V_1 = \frac{1}{3}\pi r_1^2 h,$$
$$V_2 = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2),$$
$$V_3 = \frac{1}{3}\pi h(r_2^2 + r_3^2 + r_2 r_3).$$

In the figure, $\triangle ABC \sim \triangle ADE \sim \triangle AFG$.

$$\therefore \frac{AB}{AD} = \frac{BC}{DE} \implies \frac{h}{2h} = \frac{r_1}{r_2} \implies r_2 = 2r_1$$
$$\text{and } \frac{AB}{AF} = \frac{BC}{FG} \implies \frac{h}{3h} = \frac{r_1}{r_3} \implies r_3 = 3r_1.$$

Using these values of $r_2$ and $r_3$, we get

$$V_2 = \frac{1}{3}\pi h\{r_1^2 + (2r_1)^2 + r_1(2r_1)\} = 7 \times \frac{1}{3}\pi r_1^2 h = 7V_1,$$
$$V_3 = \frac{1}{3}\pi h\{(2r_1)^2 + (3r_1)^2 + (2r_1)(3r_1)\} = 19 \times \frac{1}{3}\pi r_1^2 h = 19V_1.$$

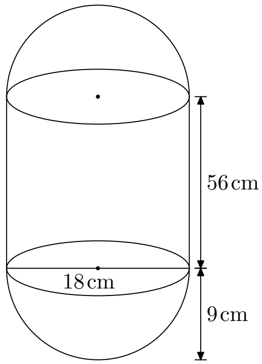
Hence, $V_1 : V_2 : V_3 = 1 : 7 : 19$. □

*Or*

**2nd problem:** Here, $h$ = the length of the cylinder = 56 cm and the radius of the cylinder, $r$

$= \text{the radius of the hemisphere, } r$

$= \frac{18}{2} = 9 \text{ cm.}$




Now, the volume of the cylinder

$$= \pi r^2 h = \frac{22}{7} \times 9^2 \times 56 = 14256 \text{ cm}^3$$

and the volume of each hemisphere

$$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 9^3 = 1527.43 \text{ cm}^3.$$

Hence, the capacity of the geyser

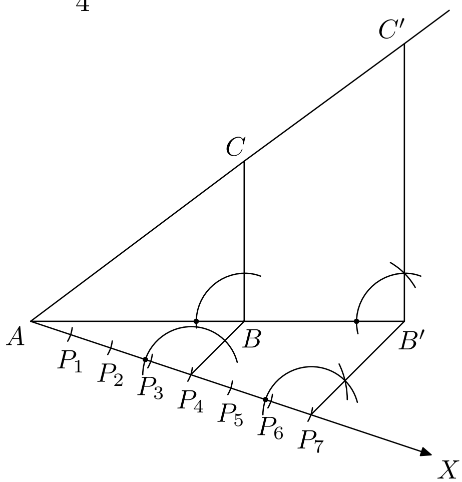
$= \text{(the volume of the cylinder)} + 2 \times \text{(the volume of a hemisphere)}$

$= 14256 + 2 \times 1527.43$

$= 17310.86 \text{ cm}^3$

$= 17.31 \text{ litres.} \quad (1000 \text{ cm}^3 = 1 \text{ litre.})$

**31.** **Required:** To construct a triangle similar to a given triangle $ABC$, the scale factor being $\frac{7}{4}$.




**Steps of construction:**

**(1)** Draw any ray $AX$ inclined to $AB$ at a certain angle, on the opposite side of $C$.

**(2)** Mark 7 points $P_1, P_2, P_3, P_4, P_5, P_6, P_7$ on $AX$ such that $AP_1 = P_1P_2 = P_2P_3 = P_3P_4 = P_4P_5 = P_5P_6 = P_6P_7$.

**(3)** Join $P_4B$ and draw $P_7B'$ parallel to $P_4B$ and meeting $AB$ produced at $B'$.

**(4)** Draw $B'C'$ parallel to $BC$, meeting $AC$ produced at $C'$.

Then $AB'C'$ is the required triangle.

**32.** Taking $a = 45$, $d_i = x_i - a$, we obtain the following table.

| Marks | $x_i$ | $d_i = x_i - 45$ | $f_i$ | $f_i d_i$ |
|---|---|---|---|---|
| $0 - 10$ | 5 | $-40$ | 15 | $-600$ |
| $10 - 20$ | 15 | $-30$ | $(35 - 15) = 20$ | $-600$ |
| $20 - 30$ | 25 | $-20$ | $(60 - 35) = 25$ | $-500$ |
| $30 - 40$ | 35 | $-10$ | $(84 - 60) = 24$ | $-240$ |
| $40 - 50$ | 45 | 0 | $(96 - 84) = 12$ | 0 |
| $50 - 60$ | 55 | 10 | $(127 - 96) = 31$ | 310 |
| $60 - 70$ | 65 | 20 | $(198 - 127) = 71$ | 1420 |
| $70 - 80$ | 75 | 30 | $(250 - 198) = 52$ | 1560 |
| | | | $N = 250$ | $\sum f_i d_i = 1350$ |

Now, $\overline{d} = \dfrac{1}{N} \sum f_i d_i = \dfrac{1}{250} \times 1350 = 5.40.$
Hence, the mean, $\overline{x} = a + \overline{d} = 45 + 5.40 = 50.40.$

The maximum frequency is 71. So, the modal class is $60 - 70$. Now, $l = 60$, $f_m = 71$, $f_1 = 31$, $f_2 = 52$, $h = 10$. Hence, the mode is given by

$$\text{mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h = 60 + \frac{71 - 31}{2 \times 71 - 31 - 52} \times 10 = 66.78.$$

## 2012

**1.** (A) 5. (By remainder theorem, the remainder is $p(2) = 5$.)

**2.** (C) $S_n = \dfrac{n}{2}[2a + (n-1)d]$.

**3.** (D) no real roots. (Discriminant $= 1^2 - 4 \times 1 \times 1 = -3 < 0$.)

**4.** (A) $\dfrac{4}{3}$. ($\sin\theta = \sqrt{1 - (3/5)^2} = 4/5$.)

**5.** (B) 1.

**6.** Yes. $0^2 = 0$, which is not positive.

**7.** By Euclid's division lemma, the possible remainders are 0, 1, 2.

**8.** The quantity $b^2-4ac$ is called the discriminant of the quadratic equation $ax^2+bx+c=0$.

**9.** Let $\alpha$ and $\frac{1}{\alpha}$ be the roots of $2x^2-5x+k=0$. Now, we have

$$\alpha \times \frac{1}{\alpha} = \frac{k}{2} \implies 1 = \frac{k}{2} \implies k = 2.$$

**10.** **Converse of Pythagoras theorem:** In a triangle, if the square of one side is equal to the sum of the squares of the remaining two sides, then the angle opposite to the first side is a right angle.

**11.** If $r$ is the radius of the circle, then $(2r)^2 = 6^2+8^2$. So, $r = 5$ cm.

**12.** The required area is $\frac{\pi r^2 \theta}{360}$.

**13.** Events are said to be mutually exclusive if the happening of one prevents the happening of all the others.

**14.** **Proof:** We have

$$\begin{aligned}
&0+0=0 && \text{(by property of additive identity)}\\
\implies &x\cdot(0+0)=x\cdot 0 && \text{(multiplying both sides by } x)\\
\implies &x\cdot 0+x\cdot 0=x\cdot 0 && \text{(by distributive law)}\\
\implies &x\cdot 0+x\cdot 0=x\cdot 0+0 && \text{(by property of additive identity)}\\
\implies &x\cdot 0=0 && \text{(by cancellation law).}
\end{aligned}$$

Thus, for any $x \in \mathbb{R}$, $x \cdot 0 = 0$. □

**15.** Since $\alpha$ and $\beta$ are the roots of $ax^2+bx+c=0$, we have

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

Now, it follows that

$$\alpha+\beta = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-2b}{2a} = -\frac{b}{a}.$$ □

**16.** The sum of the roots is $(4+\sqrt{5})+(4-\sqrt{5}) = 8$ and the product of the roots is $(4+\sqrt{5})(4-\sqrt{5}) = 11$. Therefore, the required equation is $x^2-8x+11=0$.

**17.** Here, $\cos 4\theta = \sin 6\theta = \cos(90^\circ - 6\theta)$. But $0^\circ < 6\theta < 90^\circ$. It follows that $4\theta$ and $90^\circ - 6\theta$ are acute and so $4\theta = 90^\circ - 6\theta$. Hence, $\theta = 9^\circ$.

**18.** Since there are 4 black and 5 red balls, a red ball can be drawn in 5 ways. Hence, the required probability is 5/9.

**19.** See Q.**20.**, 2010 on page 20.

**20.** The required numbers are the factors of $(408 - 23) = 385 = 5 \times 7 \times 11$ which are greater than 23. Hence, the numbers are 35, 55, 77, 385.

**21.** The given pair of equations is

$$x + y = 5, \tag{1}$$

$$2x + 3y = 12. \tag{2}$$

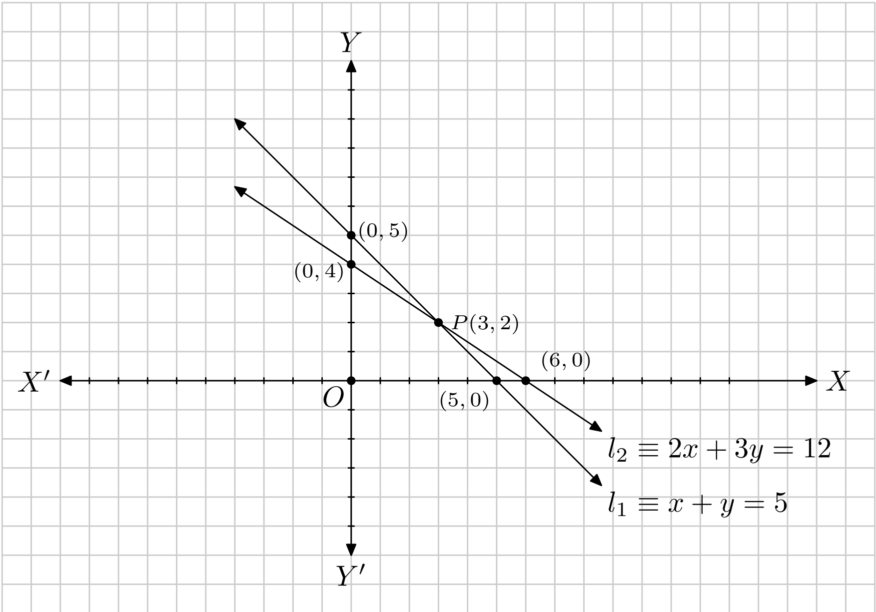
Some points satisfying (1) and (2) are respectively given below:

| $x$ | 0 | 5 | 3 |
|---|---|---|---|
| $y$ | 5 | 0 | 2 |

and

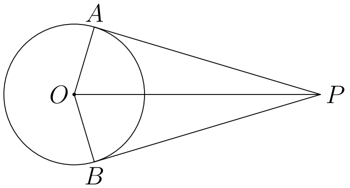
| $x$ | 0 | 6 | 3 |
|---|---|---|---|
| $y$ | 4 | 0 | 2 |

The points in the two tables are plotted and joined together in the Cartesian plane (as shown in figure) to get the straight lines $l_1$ and $l_2$ respectively. Thus, $l_1$ is the graph of $x + y = 5$ and $l_2$ is the graph of $2x + 3y = 12$.




It is seen from the figure that the two straight lines intersect at the point $P(3, 2)$. Hence, $x = 3$ and $y = 2$.

**22.** **Given:** $PA$ and $PB$ are the tangent segments drawn from an external point $P$ to a circle with centre $O$.




**To prove:** $PA = PB$.

**Construction:** Join $OP$, $OA$ and $OB$.

**Proof:** In $\triangle$s $OPA$ and $OPB$, we have

$$\begin{aligned} &\angle OAP = \angle OBP = 90^\circ, \\ &OA = OB \quad \text{(radii of the same circle)}, \\ &OP = OP \quad \text{(common)}. \\ \therefore\ &\triangle OPA \cong \triangle OPB \quad \text{(by RHS congruence)}. \\ \implies &PA = PB. \end{aligned}$$

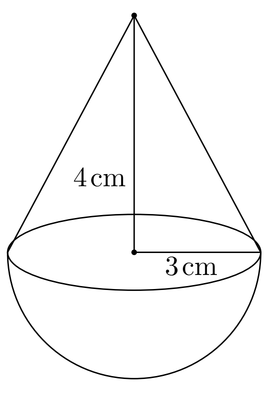
□

**23.** We proceed as follows:

$$\begin{aligned} \text{L.H.S.} &= \frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} \\ &= \frac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}^2\theta - \cot^2\theta} - \operatorname{cosec}\theta \\ &= \operatorname{cosec}\theta + \cot\theta - \operatorname{cosec}\theta \quad (\because \operatorname{cosec}^2\theta - \cot^2\theta = 1) \\ &= \cot\theta. \\ \text{R.H.S.} &= \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta} \\ &= \operatorname{cosec}\theta - \frac{\operatorname{cosec}\theta - \cot\theta}{\operatorname{cosec}^2\theta - \cot^2\theta} \\ &= \operatorname{cosec}\theta - \operatorname{cosec}\theta + \cot\theta \quad (\because \operatorname{cosec}^2\theta - \cot^2\theta = 1) \\ &= \cot\theta. \\ \therefore \text{L.H.S.} &= \text{R.H.S.} \end{aligned}$$

□

**24.** Here, the radius of the base of the cone is the same as the radius of the hemisphere, $r = 3$ cm and the height of the cone, $h = 4$ cm.




Now, the curved surface area of the cone

$$= \pi r\sqrt{r^2 + h^2} = \frac{22}{7} \times 3 \times \sqrt{(3)^2 + 4^2}$$

$$= \frac{22}{7} \times 3 \times 5 = 47.14 \text{ cm}^2$$

and the curved surface area of the hemisphere

$$= 2\pi r^2 = 2 \times \frac{22}{7} \times (3)^2 = 56.57 \text{ cm}^2.$$

Hence, the total surface area of the wooden toy

$$= 47.14 + 56.57 = 103.71 \text{ cm}^2.$$

**25.** **1st problem:** We proceed as follows:

$$\begin{aligned}
& a^3 + b^3 + c^3 - 3abc \\
&= (a+b)^3 - 3ab(a+b) + c^3 - 3abc \\
&\qquad\qquad [\because x^3 + y^3 = (x+y)^3 - 3xy(x+y)] \\
&= (a+b)^3 + c^3 - \{3ab(a+b) + 3abc\} \\
&= (a+b+c)\{(a+b)^2 - (a+b)c + c^2\} - 3ab(a+b+c) \\
&\qquad\qquad [\because x^3 + y^3 = (x+y)(x^2 - xy + y^2)] \\
&= (a+b+c)(a^2 + 2ab + b^2 - ca - bc + c^2 - 3ab) \\
&= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) \\
&= \frac{1}{2}(a+b+c)(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca) \\
&= \frac{1}{2}(a+b+c)\{(a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ca + a^2)\} \\
&= \frac{1}{2}(a+b+c)\{(a-b)^2 + (b-c)^2 + (c-a)^2\}.
\end{aligned}$$

*Or*

**2nd problem:** We have $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca.$

$$\begin{aligned}
\therefore\ & 2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4 \\
&= 4b^2c^2 - (a^4 + b^4 + c^4 - 2a^2b^2 + 2b^2c^2 - 2c^2a^2) \\
&= (2bc)^2 - (a^2 - b^2 - c^2)^2 \\
&= (2bc + a^2 - b^2 - c^2)(2bc - a^2 + b^2 + c^2) \\
&= \{a^2 - (b^2 - 2bc + c^2)\}\{(b^2 + c^2 + 2bc) - a^2\} \\
&= \{a^2 - (b-c)^2\}\{(b+c)^2 - a^2\} \\
&= (a+b-c)(a-b+c)(b+c+a)(b+c-a) \\
&= (a+b+c)(a+b-c)(b+c-a)(c+a-b).
\end{aligned}$$

**26.** Let $a_n$ be the number of cars produced in the $n^{\text{th}}$ year. Suppose the production is increased uniformly by $d$ cars every year.

It is clear that $a_1, a_2, \ldots$ form an AP with $a_3 = 600$, $a_7 = 700$ and common difference $d$. Now, we have

$$a_3 = a_1 + 2d = 600, \quad (1)$$
$$a_7 = a_1 + 6d = 700. \quad (2)$$

Subtracting (1) from (2), $4d = 100 \implies d = 25$. Now, by (1), we have

$$\therefore\ a_1 = 600 - 2 \times 25 = 550.$$

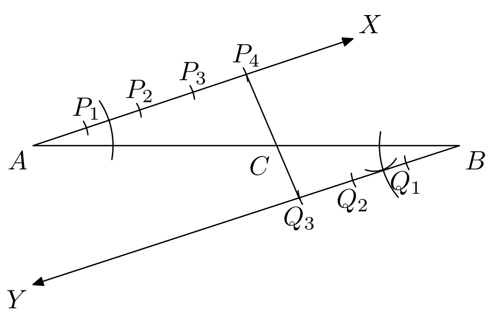
Thus, the production in the first year is 550 cars. Now, we have

$$a_{10} = a_1 + (10-1)d = 550 + 9 \times 25 = 775.$$

Hence, the production in the tenth year is 775 cars.

**27.** See Q.**28.**, 2010 on page 22.

**28.** **Diagram** (based on the properties of similarity of triangles)**:**




**Steps of construction:**

**(1)** Draw a line segment $AB$ of any length.

**(2)** Draw a ray $AX$ inclined to $AB$ at an acute angle.

**(3)** Draw a ray $BY$ parallel to $AX$ so that $\angle ABY = \angle BAX$.

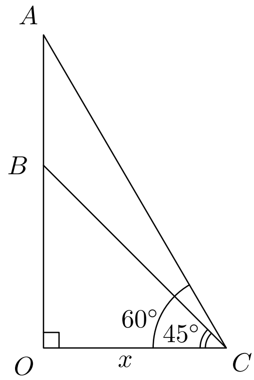
**(4)** Mark points $P_i$'s $(i = 1, 2, 3, 4)$ on $AX$ and $Q_j$'s $(j = 1, 2, 3)$ on $BY$ such that $AP_1 = P_iP_{i+1} = BQ_1 = Q_jQ_{j+1}$ for all $i = 1, 2, 3$ and $j = 1, 2$.

**(5)** Join $P_4Q_3$ intersecting $AB$ at $C$.

Then $C$ is the point on $AB$ such that $AC : CB = 4 : 3$.

**29.** **1st problem:**

Let one aeroplane be at the point $A$ vertically above the other aeroplane at $B$ and let $O$ be the point on the ground vertically bellow B. Let $C$ be the observing point on the ground. Let $OC = x$ be the distance between the aeroplanes.



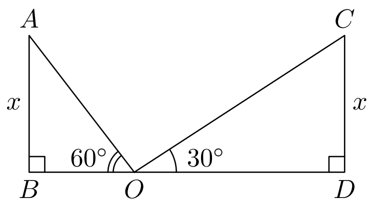

Then $OB = x \tan 45° = x$, $AC = x \sec 60° = 2x$, $OA = x \tan 60° = \sqrt{3}x$.

But $OA = 3000$ m. Hence, $x = 1000\sqrt{3} = 1730$ m.

*Or*

**2nd problem:**

Let $AB$ and $CD$ be the two towers with $B$ and $D$ on the ground. Let $O$ be the point on the ground in between them with $\angle AOB = 60°$ and $\angle COD = 30°$.




If $AB = CD = x$, then $OB = x \cot 60° = \dfrac{x}{\sqrt{3}}$ and $OD = x \cot 30° = x\sqrt{3}$ . But $BD = 60$ m. Thus, $\dfrac{x}{\sqrt{3}} + x\sqrt{3} = 60 \implies x = 15\sqrt{3} = 25.95$ m. Also, $OD = x\sqrt{3} = 15 \times 3 = 45$ m. Hence, the height of each tower is 25.95 m and the distance of the point $O$ from the foot $B$ of the tower $AB$ is 45 m.

**30.** Here, $h = 20$ cm, $r_1 = 10$ cm, $r_2 = 25$ cm. So, the slant height, $l = h^2 + (r_2 - r_1)^2 = 25$ cm. Now, the volume of the frustum $= \frac{1}{3} \times \frac{22}{7} \times \left(10^2 + 10 \times 25 + 25^2\right) = 11392.86$ cm$^3$ $= 11.39$ litres. Hence, the cost of milk which can fill the container = Rs $20 \times 11.39$ = Rs 227.80. Also, the total surface area of the frustum $= \pi[l(r_1 + r_2) + r_1^2 + r_2^2] = 5028.57$ cm$^2$. Hence, the cost of metal sheet = Rs $\frac{10}{100} \times 5028.57$ = Rs 502.86.

**31.** **1st problem: AAA similarity:** If the corresponding angles of two triangles are equal, then the triangles are similar.

**Given:** Two $\triangle$s $ABC$ and $DEF$ in which $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$.

**To prove:** $\triangle ABC \sim \triangle DEF$.

**Construction:** Take points $M$ and $N$ on $AB$ and $AC$ respectively such that $AM = DE$ and $AN = DF$. Join $MN$.

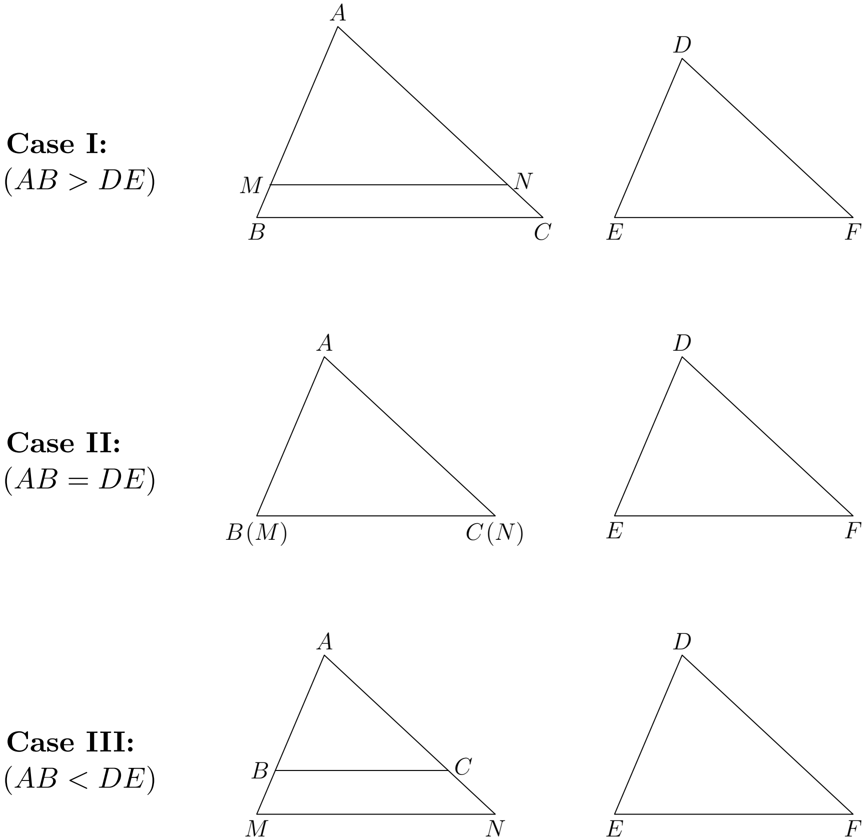



Figure 1.1: Similarity of triangles.

**Proof:** Three cases arise.

**Case I:** $AB > DE$. Here, $M$ lies in $AB$. Now, in $\triangle$s $AMN$ and $DEF$,

$$AM = DE, \quad AN = DF \quad \text{and} \quad \angle A = \angle D.$$

By SAS congruence, $\triangle AMN \cong \triangle DEF$. So, $\angle AMN = \angle DEF$. But $\angle DEF = \angle ABC$. Therefore, $\angle AMN = \angle ABC$. It implies that $MN \| BC$. Now, by a deduction from Thales' theorem, we have

$$\frac{AB}{AM} = \frac{AC}{AN} \implies \frac{AB}{DE} = \frac{AC}{DF}.$$

Similarly, it can be shown that

$$\frac{AB}{DE} = \frac{BC}{EF}.$$

Thus, we have

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}.$$

**Case II:** $AB = DE$. Here, $M$ coincides with $B$. Then, by ASA congruence, we have

$$\begin{aligned} &\triangle ABC \cong \triangle DEF \\ \implies &AB = DE,\ BC = EF,\ AC = DF \\ \implies &\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}. \end{aligned}$$

**Case III:** $AB < DE$. Here, $M$ lies on $AB$ produced. As in Case I, it can be shown that $BC \| MN$ and ultimately, we obtain

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}.$$

Thus, in all possible cases, we get

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF},$$

i.e., the corresponding sides of $\triangle ABC$ and $\triangle DEF$ are proportional. As the corresponding angles are also equal, we can conclude that $\triangle ABC \sim \triangle DEF$. □

*Or*

**2nd problem: SSS similarity:** If the corresponding sides of two triangles are in the same ratio, then the triangles are similar.

**Given:** Two triangles $ABC$ and $DEF$ such that

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}.$$

**To prove:** $\triangle ABC \sim \triangle DEF$.

**Construction:** Take points $M$ and $N$ on $AB$ and $AC$ respectively such that $AM = DE$ and $AN = DF$. Join $MN$. See diagram on page 44.

**Proof:** Three cases arise.

**Case I:** $AB > DE$. Here, $M$ lies in $AB$. Now, we have

$$\begin{aligned} &\frac{AB}{DE} = \frac{AC}{DF} \\ \implies &\frac{AB}{AM} = \frac{AC}{AN} \\ \implies &\frac{AB}{AM} - 1 = \frac{AC}{AN} - 1 \end{aligned}$$

$$\implies \frac{AB - AM}{AM} = \frac{AC - AN}{AN}$$
$$\implies \frac{MB}{AM} = \frac{NC}{AN}$$
$$\implies \frac{AM}{AB} = \frac{AN}{AC}.$$

Therefore, by the converse of basic proportionality theorem, $MN \| BC$. It implies that $\angle AMN = \angle ABC$ and $\angle ANM = \angle ACB$. So, by AA similarity, $\triangle ABC \sim \triangle AMN$.

$$\therefore \frac{AB}{AM} = \frac{BC}{MN} \implies \frac{AB}{DE} = \frac{BC}{MN}. \quad (1)$$

But

$$\frac{AB}{DE} = \frac{BC}{EF}. \quad (2)$$

From (1) and (2), we have

$$\frac{BC}{MN} = \frac{BC}{EF} \implies MN = EF.$$

By SSS congruence, $\triangle AMN \cong \triangle DEF$. Thus, $\triangle AMN$ and $\triangle DEF$ are equiangular. But $\triangle ABC \sim \triangle AMN$ and so they are also equiangular. So, $\triangle ABC$ and $\triangle DEF$ are equiangular. As the corresponding sides of $\triangle ABC$ and $\triangle DEF$ are proportional, we can conclude that $\triangle ABC \sim \triangle DEF$.

**Case II:** $AB = DE$. Here, $M$ coincides with $E$. Now, we have

$$1 = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \implies AB = DE,\ BC = EF,\ AC = DF.$$

So, by SSS congruence, $\triangle ABC \cong \triangle DEF$. Hence, $\triangle ABC \sim \triangle DEF$.

**Case III:** $AB < DE$. Here, $M$ lies on $AB$ produced. As in Case I, it can be shown that $BC \| MN$ and ultimately, we obtain $\triangle ABC \sim \triangle DEF$.

Thus, in all possible cases, $\triangle ABC \sim \triangle DEF$. □

**32.** Taking $a = 55$, $h = 10$, $u_i = \dfrac{x_i - a}{h} = \dfrac{x_i - 55}{10}$, we obtain the following table.

| Marks | $x_i$ | $u_i = \frac{x_i-55}{10}$ | $f_i$ | c.f. | $f_i u_i$ |
|---|---|---|---|---|---|
| $0-10$ | 5 | $-5$ | $(80-77)=3$ | 3 | $-15$ |
| $10-20$ | 15 | $-4$ | $(77-72)=5$ | 8 | $-20$ |
| $20-30$ | 25 | $-3$ | $(72-65)=7$ | 15 | $-21$ |
| $30-40$ | 35 | $-2$ | $(65-55)=10$ | 25 | $-20$ |
| $40-50$ | 45 | $-1$ | $(55-43)=12$ | 37 | $-12$ |
| $50-60$ | 55 | 0 | $(43-28)=15$ | 52 | 0 |
| $60-70$ | 65 | 1 | $(28-16)=12$ | 64 | 12 |
| $70-80$ | 75 | 2 | $(16-10)=6$ | 70 | 12 |
| $80-90$ | 85 | 3 | $(10-8)=2$ | 72 | 6 |
| $90-100$ | 95 | 4 | $(8-0)=8$ | 80 | 32 |
| | | | $N=80$ | | $\sum f_i u_i = -26$ |

Now, $\overline{u} = \frac{1}{N}\sum f_i u_i = \frac{1}{80} \times (-26) = -0.325$.
Hence, the mean mark, $\overline{x} = a + h\overline{u} = 55 + 10 \times (-0.325) = 51.75$.

**Median:** Here, $\frac{N}{2} = \frac{80}{2} = 40$. The cumulative frequency just greater than 40 is 52. So, the corresponding class $50-60$ is the median class. Now, $l = 50,\ f = 15,\ c = 37$.

$$\therefore \text{ median} = l + \frac{\frac{N}{2} - c}{f} \times h = 50 + \frac{50-37}{15} \times 10 = 50 + 5.2 = 55.2.$$

## 2013

**1.** (B) $p(a) = 0$. (Factor theorem.)

**2.** (A) $-\frac{b}{a}$.

**3.** (C) 13. (It is the number of terms of the AP: $14, 21, 28, \ldots, 98$.)

**4.** (B) $11^\circ$. ($\because\ \cot 3\theta = \tan(90^\circ - 3\theta)$.)

**5.** (D) $\frac{\pi r^2 \theta}{360}$.

**6.** An algebraic expression which remains unchanged under the cyclical replacement of the letters involved is called a cyclic expression.

**7.** $2013 = 3 \times 11 \times 61$.

**8.** Since the discriminant is 0, the roots are real and equal.

**9.** Since $a_n = 3n - 2$, $a_{n+1} = 3n + 1$. So, the common difference $= a_{n+1} - a_n = 3$.

**10.** **Pythagoras theorem:** In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

**11.** The volume of a frustum is $\frac{1}{3}\pi h(r_1^2 + r_1 r_2 + r_2^2)$, where $h$ is the height of the frustum, $r_1$ and $r_2$ are the radii of the bases of the frustum.

**12.** Here, $r = 6$ cm, $\theta = 30°$. So, the arc length $= \dfrac{\theta}{360} \times 2\pi r = \dfrac{30}{360} \times 2 \times 3.14 \times 6 = 3.14$ cm.

**13.** **Random experiment:** A random experiment is an experiment whose result cannot be uniquely predicted even if the previous results of the same experiment conducted under similar conditions are all known.

**14.** Every integer $a$ is of the form $2q$ or $2q + 1$. So, $a^2 = 4k$, where $k = q^2$, or $a^2 = 4k + 1$, where $k = q^2 + q$.

**15.** See Q.**15.**, 2011 on page 30.

**16.** Let $\alpha$ and $2\alpha$ be the roots of $x^2 - px + q = 0$. Then

$$\alpha + 2\alpha = p \implies \alpha = \frac{p}{3} \quad (1)$$

$$\text{and } \alpha \cdot 2\alpha = q$$

$$\implies 2\left(\frac{p}{3}\right)^2 = q \qquad \text{(by (1))}$$

$$\implies 2p^2 = 9q. \qquad \square$$

**17.** L.H.S. $= \cos^2\theta - \sin^2\theta = \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \dfrac{\frac{\cos^2\theta}{\sin^2\theta} - 1}{\frac{\cos^2\theta}{\sin^2\theta} + 1} = \dfrac{\cot^2\theta - 1}{\cot^2\theta + 1} =$ R.H.S. $\square$

**18.** The volume of the cone is $\dfrac{1}{3}\pi \times 3^2 \times 12 = 36\pi$ cm$^3$. If $r$ is the radius of the sphere, then $\dfrac{4}{3}\pi r^3 = 36\pi \implies r = 3$. Hence, the radius of the sphere is 3 cm.

**19.** See Q.**20.**, 2011 on page 31.

**20.** We proceed as follows:

$$x\left(y^2 + z^2\right) + y\left(z^2 + x^2\right) + z\left(x^2 + y^2\right) + 3xyz$$

$$= xy^2 + z^2x + yz^2 + x^2y + zx^2 + y^2z + 3xyz$$
$$= \left(xy^2 + xyz + x^2y\right) + \left(z^2x + xyz + zx^2\right) + \left(yz^2 + xyz + y^2z\right)$$
$$= xy(y + z + x) + zx(z + y + x) + yz(z + x + y)$$
$$= (x + y + z)(xy + yz + zx).$$

**21.** The given pair of equations is

$$x + 2y = 7, \quad (1)$$
$$2x - y = 4. \quad (2)$$

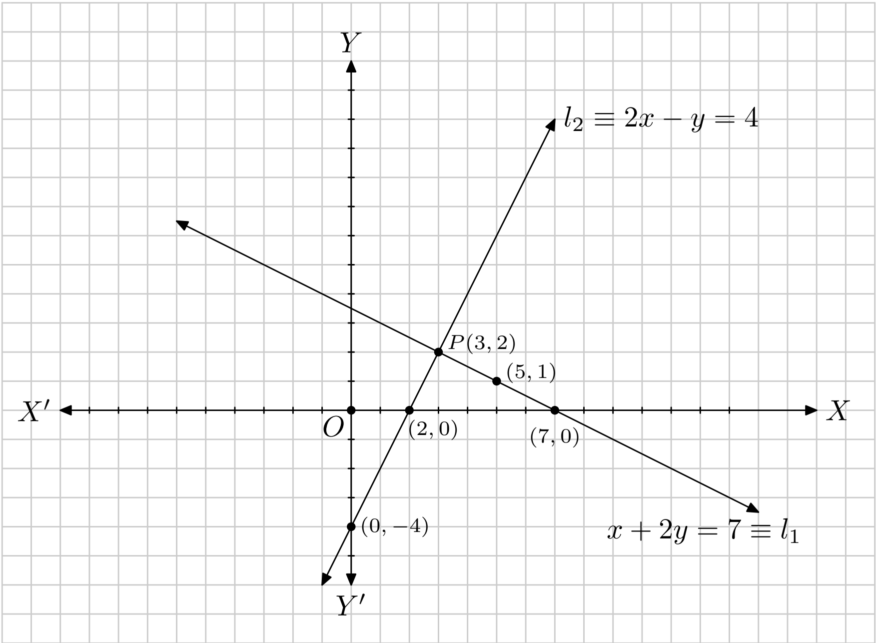
Some points satisfying equations (1) and (2) are respectively given below:

| $x$ | 5 | 3 | 7 |
|---|---|---|---|
| $y$ | 1 | 2 | 0 |

and

| $x$ | 0 | 2 | 3 |
|---|---|---|---|
| $y$ | $-4$ | 0 | 2 |

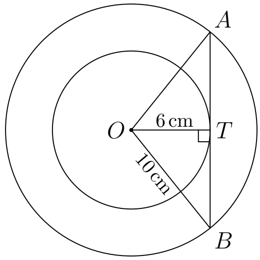
The points in the two tables are plotted and joined together in the Cartesian plane (as shown in figure) to get the straight lines $l_1$ and $l_2$ respectively. Thus, $l_1$ is the graph of $x + 2y = 7$ and $l_2$ is the graph of $2x - y = 4$.




It is seen from the figure that the two straight lines intersect at the point $P(3, 2)$. Hence, the solution of the given pair of equations is given by $x = 2$ and $y = 3$.

**22.** Let $O$ be the common centre of the concentric circles and $AB$ be the chord of the larger circle which touches the smaller circle at $T$. Here, $OT = 6$ cm and $OA = OB = 10$ cm.

Since $AB$ is a tangent to the smaller circle, $OT \perp AB$. Thus, $OBT$ forms a right triangle. Now, by Pythagoras theorem,




$$\begin{aligned} & OT^2 + TB^2 = OB^2 \\ \implies & TB^2 = OB^2 - OT^2 = 10^2 - 6^2 = 64 \\ \implies & TB = \sqrt{64} = 8 \text{ cm.} \end{aligned}$$

Now, in $\triangle$s $ATO$ and $BTO$, we have

$$\begin{aligned} & OA = OB \quad \text{(radii of the same circle)}, \\ & OT = OT \quad \text{(sides common to both } \triangle\text{s)}, \\ & \angle OTA = OTB \quad (90^\circ \text{ each}). \\ \therefore\ & \triangle ATO \cong \triangle BTO \quad \text{(by RHS congruence)}. \end{aligned}$$

It follows that $AT = BT$ cm. Hence, $AB = AT + BT = 2AT = 2 \times 8 = 16$ cm.

**23.** **Trigonometric ratios of** $60^\circ$**:** Let us consider an equilateral triangle $ABC$. Then $\angle A = \angle B = \angle C = 60^\circ$. From $A$ we draw $AD$ perpendicular to $BC$ meeting $BC$ at $D$.

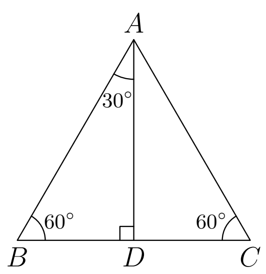
Then, by RHS congruence theorem, we have $\triangle ABD \cong \triangle ACD$. So,

$$BD = CD \text{ and } \angle BAD = \angle CAD.$$

Consequently, we have

$$BD = CD = \frac{1}{2}BC \quad \text{and} \quad \angle BAD = \angle CAD = \frac{1}{2}\angle A = \frac{1}{2} \times 60^\circ = 30^\circ.$$

We know that $ABD$ is a right triangle, right angled at $D$. Here, $\angle ABD = 60^\circ$. Suppose $AB = 2k$ for some positive number $k$. Then $BD = \frac{1}{2}BC = \frac{1}{2}AB = k$. Now, by applying Pythagoras theorem on $\triangle ABD$, we have




$$\begin{aligned} & AD^2 = AB^2 - BD^2 = (2k)^2 - k^2 = 3k^2 \\ \implies & AD = \sqrt{3}k. \end{aligned}$$

In right triangle $ABD$, by definitions of the trigonometric ratios, we have

$$\sin 60^\circ = \frac{AD}{AB} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}, \qquad \operatorname{cosec} 60^\circ = \frac{AB}{AD} = \frac{2k}{\sqrt{3}k} = \frac{2}{\sqrt{3}}$$
$$\cos 60^\circ = \frac{BD}{AB} = \frac{k}{2k} = \frac{1}{2}, \qquad \sec 60^\circ = \frac{AB}{BD} = \frac{2k}{k} = 2,$$
$$\tan 60^\circ = \frac{AD}{BD} = \frac{\sqrt{3}k}{k} = \sqrt{3}, \qquad \cot 60^\circ = \frac{BD}{AD} = \frac{k}{\sqrt{3}k} = \frac{1}{\sqrt{3}}.$$

**24.** The sample space $S$ is given by

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}.$$

Now, $\{HHT, HTH, THH\}$ is the event that the head appears exactly twice. Hence, the required probability $= \frac{3}{8}$.

**25. 1st problem: Proof:** If $x \neq 0$, then $\frac{1}{x} \in \mathbb{R}$. Thus,

$$\begin{aligned} xy = 0 &\implies \frac{1}{x}(xy) = \frac{1}{x} \cdot 0 \\ &\implies \left(\frac{1}{x} \cdot x\right) y = 0 \\ &\implies 1 \cdot y = 0 \\ &\implies y = 0. \end{aligned}$$

Similarly, if $y \neq 0$, then $x = 0$. Hence, $xy = 0 \implies x = 0$ or $y = 0$. □

*Or*

**2nd problem: Proof:** Suppose $|x - y| < \delta$. Then

$$x - y \leq |x - y| < \delta \implies x - y < \delta \implies x < y + \delta \tag{1}$$

and

$$\begin{aligned} &-(x - y) \leq |x - y| < \delta \\ \implies &-x + y < \delta \\ \implies &y < x + \delta \\ \implies &y - \delta < x. \end{aligned} \tag{2}$$

Combining (1) and (2), we get $y - \delta < x < y + \delta$.

Conversely, suppose $y-\delta < x < y+\delta$. Then

$$y-\delta < x \implies y-x<\delta \implies -(x-y)<\delta$$

and $\quad x<y+\delta \implies x-y<\delta.$
Thus, the greater of $x-y$ and $-(x-y)$ is less than $\delta$ as both $x-y$ and $-(x-y)$ are less than $\delta$. Hence, $|x-y|<\delta$. $\square$

**26.** Let $x$ be the number of benches and $y$ be the number of students. By the given conditions, we get

$$4(x-5)=y \implies 4x-y=20 \tag{1}$$
$$\text{and } 3x+4=y \implies 3x-y=-4. \tag{2}$$

Subtracting (2) from (1), we get

$$4x-3x=20+4 \implies x=24.$$

Using this value of $x$ in (1), we get

$$4\times 24-y=20 \implies y=76.$$

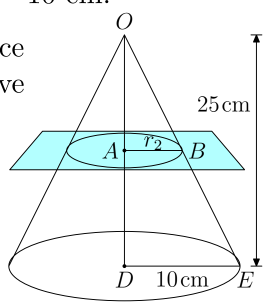
Hence, the number of benches is 24 and the number of students is 76.

**27.** See 2nd problem, Q.**28.**, 2010 on page 24.

**28.** See Q.**31.**, 2011 on page 36.

**29.** See 1st problem, Q.**32.**, 2010 on page 27. Replace $\alpha$, $\beta$ and $h$ by their appropriate values ($\alpha=60°$, $\beta=30°$, $h=10$ m). The height of the tower is 30 m.

**30.** **1st problem:** Here, the height of the original cone, $h_1=25$ cm and the radius of the base of the original cone, $r_1=10$ cm.




Let $r_2$ cm be the radius of the cross-section. Since the area of the cross-section is 154 cm$^2$, we have

$$\pi r_2^2=154 \implies \frac{22}{7}\times r_2^2=154.$$

$\therefore\ r_2^2=49 \implies r_2=7$ cm.

Since $\triangle OAB \sim \triangle ODE$, we have

$$\frac{OA}{OD}=\frac{r_2}{r_1} \implies \frac{OA}{25}=\frac{7}{10} \implies OA=17.5 \text{ cm}.$$

Therefore, the distance of the plane from the base of the cone
$= 25 - 17.5$
$= 7.5$ cm.

*Or*

**2nd problem:** The height of the conical vessel is 6 cm and the radius of the conical vessel is 4 cm. Thus, the volume of the conical vessel, $V_{co} = \frac{1}{3} \times \frac{22}{7} \times 4^2 \times 6 = \frac{704}{7}$ cm$^3$.

So, the volume of water flowed out of the vessel, $V_w$

$$= \frac{1}{10} \times V_{co} = \frac{1}{10} \times \frac{704}{7} = \frac{704}{70} \text{ cm}^3.$$

Now, the radius of a spherical lead shot = 0.2 cm.

So, the volume of a spherical lead shot, $V_s$

$$= \frac{4}{3} \times \frac{22}{7} \times (0.2)^3 = \frac{0.704}{21} \text{ cm}^3.$$

Hence, the number of the lead shots dropped into the vessel

$$= \frac{V_w}{V_s} = \frac{\left(\frac{704}{70}\right)}{\left(\frac{0.704}{21}\right)} = \frac{3000}{10} = 300.$$

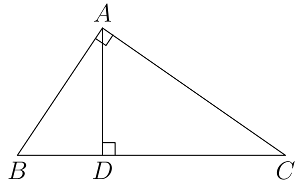
**31. 1st problem:** See 1st problem, Q.**30.**, 2010 on page 25.

*Or*

**2nd problem:**

**Given:** $ABC$ is a right triangle right angled at $A$ and $AD \perp BC$.

**To prove:** **(i)** $\triangle DBA \sim \triangle ABC$,
**(ii)** $\triangle DAC \sim \triangle ABC$, **(iii)** $\triangle DBA \sim \triangle DAC$.




**Proof:** **(i)** In $\triangle$s $DBA$ and $ABC$,

$\angle DBA = \angle ABC$ (same angle),
$\angle ADB = \angle CAB = 90°$.
$\therefore \triangle DBA \sim \triangle ABC$ (AA similarity). □

**(ii)** In $\triangle$s $DAC$ and $ABC$,

$\angle DCA = \angle ACB$ (same angle),

$$\angle ADC = \angle CBAC = 90^\circ.$$

$$\therefore\ \triangle DAC \sim \triangle ABC \quad \text{(AA similarity)}. \qquad \square$$

**(iii)** In $\triangle$s $DBA$ and $DAC$,

$$\angle ADB = \angle CDA = 90^\circ,$$
$$\angle DBA = 90^\circ - \angle BAD = \angle DAC.$$
$$\therefore\ \triangle DBA \sim \triangle DAC \quad \text{(AA similarity)}. \qquad \square$$

**32.** We modify the given data as follows:

| Expenditure of water (in Rs) | Mid value $x_i$ | No. of families $f_i$ | Cumulative frequency | $f_i x_i$ |
|---|---|---|---|---|
| 30 – 40 | 35 | 12 | 12 | 420 |
| 40 – 50 | 45 | 18 | 30 | 810 |
| 50 – 60 | 55 | 20 | 50 | 1100 |
| 60 – 70 | 65 | 15 | 65 | 975 |
| 70 – 80 | 75 | 12 | 77 | 900 |
| 80 – 90 | 85 | 11 | 88 | 935 |
| 90 – 100 | 95 | 6 | 94 | 570 |
| 100 – 110 | 105 | 4 | 98 | 420 |
| 110 – 120 | 115 | 2 | $100 = N$ | 230 |
| | | $N = 100$ | $\sum_{i=1}^{9} f_i x_i = 6360$ | |

**Mean:** The mean monthly expenditure of the families on water is

$$\overline{x} = \frac{1}{N}\sum_{i=1}^{9} f_i x_i = \frac{1}{100} \times 6360 = 63.60.$$

**Quartiles:** Here, $N = 100 \implies \dfrac{N}{4} = \dfrac{100}{4} = 25$. The cumulative frequency just greater than $\dfrac{N}{4} = 25$ is 30. So, its corresponding class $40 - 50$ is the $Q_1$ class, and $l = 40$, $f = 18$, $c = 12$.

$$\therefore\ Q_1 = l + \frac{\frac{N}{4} - c}{f} \times h = 40 + \frac{25 - 12}{18} \times 10 = 47.22.$$

Again, $\dfrac{3N}{4} = \dfrac{3 \times 100}{4} = 75$. Now, the corresponding frequency just greater than 75 is 77. So, its corresponding class $70 - 80$ is the $Q_3$ class, and $l = 70$, $f = 12$, $c = 65$.

$$\therefore\ Q_3 = l + \frac{\frac{3N}{4} - c}{f} \times h = 70 + \frac{75 - 65}{12} \times 10 = 78.33.$$

## 2014

**1.** (C) $-\frac{5}{2}$. (Note that $|x + 5| = -x$ implies either $x + 5 = -x$ or $x + 5 = -(-x)$, the latter being impossible.)

**2.** (D) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

**3.** (A) $-5$. (Here, $d = \frac{a_{22} - a_{15}}{22 - 15} = \frac{-35}{7} = -5$.)

**4.** (B) 38.5. (Here, $2\pi r = 44 \implies r = 7$ cm. Area of the circle $= 154$ cm$^2$. Area of a quadrant $= \frac{154}{4} = 38.5$ cm$^2$.)

**5.** (C) $P(A) \cdot P(B)$.

**6.** **Euclid's division lemma:** Let $a$ and $b$ be any two integers and $b > 0$. Then there exist unique integers $q$ and $r$ such that $a = bq + r$ and $0 \leq r < b$.

**7.** By remainder theorem, the required remainder is $2 \times 1^2 - 3 \times 1 + 2 = 1$.

**8.** **Arithmetic progression:** A sequence $a_1, a_2, \ldots, a_n, \ldots$ is called an arithmetic progression (AP) if the difference $(a_{n+1} - a_n)$ is constant for all $n \in \mathbb{N}$.

**9.** The sequence is $4, 7, 12, \ldots$. Since $7 - 4 \neq 12 - 7$, it is not an AP.

**10.** We see that $7^2 + 24^2 = 25^2$. So, by the converse of Pythagoras theorem, the triangle is a right triangle.

**11.** The Pythagorean relation between $\sec A$ and $\tan A$ is $\tan^2 A + 1 = \sec^2 A$, where $A \neq 90°$.

**12.** The volume of a frustum is $\frac{1}{3}\pi h(r_1^2 + r_1 r_2 + r_2^2)$, where $h$ is the height of the frustum, $r_1$ and $r_2$ are the radii of the bases of the frustum.

**13.** Events are said to be equally likely if there is no valid reason to say that one event has more chance to occur than the others.

**14.** **Proof:** Since $x \in \mathbb{R}$, by existence of additive inverse, $-x \in \mathbb{R}$. Thus,

$$\begin{aligned} & x+y=x+z \\ \Longrightarrow\ & (-x)+(x+y)=(-x)+(x+z) \quad \text{(adding } -x \text{ to both sides)} \\ \Longrightarrow\ & (-x+x)+y=(-x+x)+z \quad \text{(by associativity of addition)} \\ \Longrightarrow\ & 0+y=0+z \quad \text{(by property of additive inverse)} \\ \Longrightarrow\ & y=z \quad \text{(by property of the additive identity).} \end{aligned}$$

This completes the proof. □

**15.** Let us assume that $ax^2+bx+c=0$ is the required equation. Then $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$. Now, we have

$$\begin{aligned} & ax^2+bx+c=0 \\ \Longrightarrow\ & x^2+\frac{b}{a}x+\frac{c}{a}=0 \\ \Longrightarrow\ & x^2-\left(-\frac{b}{a}\right)x+\frac{c}{a}=0 \\ \Longrightarrow\ & x^2-(\alpha+\beta)x+\alpha\beta=0. \end{aligned}$$

Thus, the required quadratic equation is $x^2-(\alpha+\beta)x+\alpha\beta=0$.

**16.** Here, $a=20$, $d=17-20=-3$. So, the sum of the first 25 terms of the AP is given by

$$S_{25}=\frac{25}{2}[2\times 20+(25-1)\times(-3)]=-400.$$

**17.** Here, $\cos A=\frac{4}{5}$. Now, we have

$$\begin{aligned} & \cos^2 A+\sin^2 A=1 \\ \Longrightarrow\ & \left(\frac{4}{5}\right)^2+\sin^2 A=1 \\ \Longrightarrow\ & \sin^2 A=1-\frac{16}{25}=\frac{9}{25} \\ \Longrightarrow\ & \sin A=\frac{3}{5} \quad \text{(neglecting the negative value).} \end{aligned}$$

Again, $\tan A=\frac{\sin A}{\cos A}=\frac{(3/5)}{(4/5)}=\frac{3}{4}$.

**18.** There are six possible outcomes in throwing a die. Out of these six outcomes, only two outcomes, namely 5 and 6 are favourable to the event of getting a number greater than 4. Hence, the required probability $= \frac{2}{6} = \frac{1}{3}$.

**19.** We proceed as follows:

$$\begin{aligned}
&a^2(b+c) + b^2(c+a) + c^2(a+b) + 2abc \\
&= a^2(b+c) + b^2c + ab^2 + c^2a + bc^2 + 2abc \\
&= a^2(b+c) + a(b^2 + c^2 + 2bc) + (b^2c + bc^2) \\
&\qquad\qquad\qquad\qquad\text{(arranging in descending powers of } a) \\
&= a^2(b+c) + a(b+c)^2 + bc(b+c) \\
&= (b+c)[a^2 + a(b+c) + bc] \\
&= (b+c)[b(c+a) + a(c+a)] \quad \text{(arranging in descending powers of } b) \\
&= (b+c)(c+a)(b+a) \\
&= (a+b)(b+c)(c+a).
\end{aligned}$$

**20.** The canonical decompositions of 420, 504 and 924 are given by

$$\begin{aligned}
420 &= 2^2 \times 3 \times 5 \times 7, \\
504 &= 2^3 \times 3^2 \times 7, \\
924 &= 2^2 \times 3 \times 7 \times 11.
\end{aligned}$$

Hence, the required HCF $= 2^2 \times 3 \times 7 = 84$ and the LCM $= 2^3 \times 3^2 \times 5 \times 7 \times 11 = 27720$.

**21.** The given pair of equations is

$$4x + 3y = 12, \qquad (1)$$
$$5x - 3y = 15. \qquad (2)$$

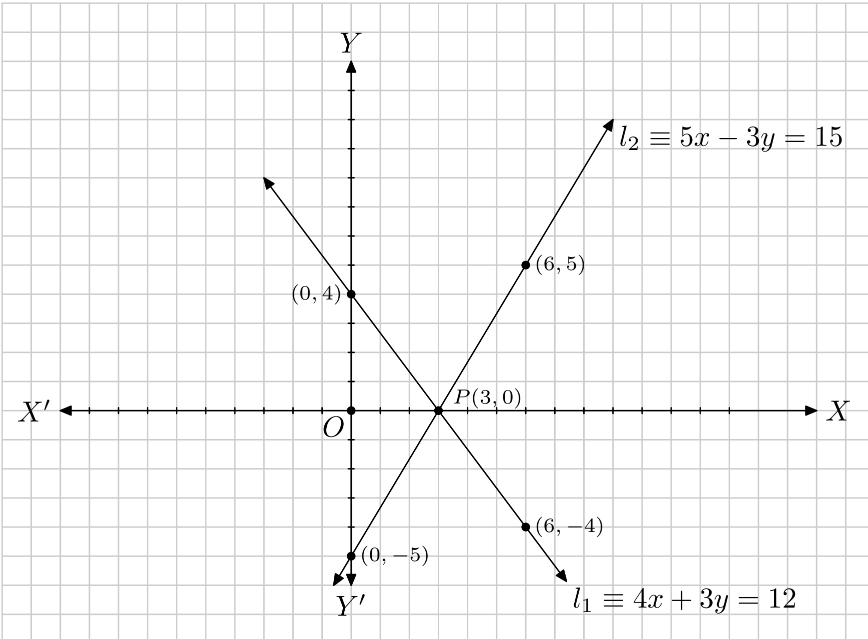
Some points satisfying equations (1) and (2) are respectively given below:

| $x$ | 0 | 3 | 6 |
|---|---|---|---|
| $y$ | 4 | 0 | $-4$ |

and

| $x$ | 0 | 3 | 6 |
|---|---|---|---|
| $y$ | $-5$ | 0 | 5 |

The points in the two tables are plotted and joined together in the Cartesian plane (as shown in figure) to get the straight lines $l_1$ and $l_2$ respectively. Thus, $l_1$ is the graph of $4x + 3y = 12$ and $l_2$ is the graph of $5x - 3y = 15$.



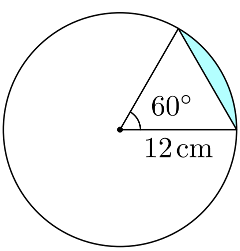

It is seen from the figure that the two straight lines intersect at the point $P(3,0)$. Therefore, the coordinates of $P$ will satisfy both the equations (1) and (2). Hence, the solution of the given pair of equations is given by $x = 3$ and $y = 0$.

**22.** See Q.**23.**, 2010 on page 21.

**23.** See Q.**24.**, 2010 on page 21.

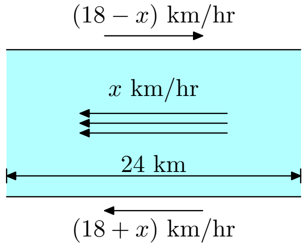
**24.** Here, the radius of the circle, $r = 12$ cm and the sectorial angle, $\theta = 60°$. Then the area of the minor segment is

$$\begin{aligned}\frac{r^2}{2}\left(\frac{\theta\pi}{180} - \sin\theta\right) &= \frac{12^2}{2}\left(\frac{60 \times 3.14}{180} - \sin 60°\right)\\ &= 72\left(\frac{3.14}{3} - \frac{\sqrt{3}}{2}\right)\\ &= 72\left(\frac{3.14}{3} - \frac{1.73}{2}\right)\\ &= 13.08 \text{ cm}^2.\end{aligned}$$




**25.** See Q.**20.**, 2010 on page 20.

**26.** **1st problem:** Let $x$ km/hr be the speed of the stream. Then the upstream speed of the streamer is $(18-x)$ km/hr and the downstream speed of the streamer is $(18+x)$ km/hr.




So, the time taken by the streamer to go 24 km upstream is $24/(18-x)$ hr and the time taken by the streamer to return downstream to the same spot is $24/(18+x)$ hr. By the given conditions, we get

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$
$$\implies 24\{(18+x)-(18-x)\} = (18-x)(18+x) = 18^2 - x^2$$
$$\implies x^2 + 48x - 18^2 = 0$$
$$\implies x^2 - 6x + 54x - 54 \times 6 = 0$$
$$\implies x(x-6) + 54(x-6) = 0$$
$$\implies (x-6)(x+54) = 0.$$
$$\therefore x = 6 \text{ or } x = -54.$$

Since the speed of the stream is nonnegative, $x = -54$ is neglected and so $x = 6$. Hence, the speed of the stream is 6 km/hr.

**2nd problem:** Let $x$ be the digit in the units place and $y$ be the digit in tens place. Then the number is $10y + x$.
By the given conditions, we get

$$xy = 20 \qquad (1)$$
$$\text{and} \quad 10y + x + 9 = 10x + y$$
$$\implies 9y - 9x + 9 = 0$$
$$\implies y - x + 1 = 0$$
$$\implies y = x - 1. \qquad (2)$$

Putting this value of $y$ in (1), we get

$$x(x-1) = 20$$
$$\implies x^2 - x - 20 = 0$$
$$\implies (x-5)(x+4) = 0.$$
$$\therefore x = 5 \text{ or } x = -4.$$

Since $x$ is a digit, $x$ cannot be negative and hence $x = -4$ is neglected. Therefore, $x = 5$ and by (2), $y = 5 - 1 = 4$. Hence, the number is $10 \times 4 + 5 = 45$.

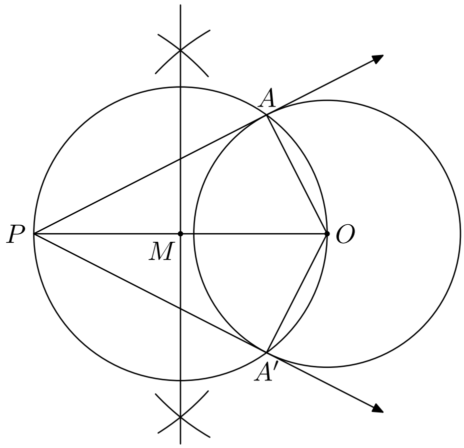
**27.** Let $k:1$ be the required ratio. Then the coordinates of the point which divides the line joining the points $(-2,3)$ and $(3,8)$ in the ratio $k:1$ are $\left(\dfrac{3k-2}{k+1}, \dfrac{8k+3}{k+1}\right)$. But this point lies on the $y$-axis, so the abscissa is 0.

$$\therefore\ \frac{3k-2}{k+1} = 0 \implies 3k-2=0 \implies k=\frac{2}{3}.$$

Hence, the required ratio is $2:3$ and the point of division is

$$\left(\frac{3k-2}{k+1}, \frac{7k-3}{k+1}\right) = \left(0, \frac{7\times\frac{2}{3}-3}{\frac{2}{3}+1}\right) = (0,1)\,.$$

**28.** We construct a pair of tangents to a circle from an external point $P$ as follows:




**Given:** A circle with centre $O$ and an external point $P$.

**Required:** To construct a pair of tangents from $P$ to the circle.

**Steps of construction:**

**(1)** Join $PO$.
**(2)** Bisect $PO$ at $M$.
**(3)** Draw a circle with centre $M$ and radius $MP$ (or $MO$) cutting the given circle at $A$ and $A'$.
**(4)** Draw rays $PA$ and $PA'$.

Thus, $PA$ and $PA'$ is the required pair of tangents.

**29.** **1st problem:** See 2nd problem, Q.**32.**, 2010 on page 27.

*Or*

**2nd problem:** See 1st problem, Q.**32.**, 2010 on page 27.

**30.** Let $r$ cm be the radius of the base of the cylinder. Since the volume of the sphere is the same as the volume of the cylinder, we have

$$\frac{4}{3}\pi \times 6^3 = \pi r^2 \times 32 \implies r = 3.$$

Hence, the total surface area of the cylinder is $2\pi r(r+h) = 2 \times 3.14 \times 3 \times 35 = 659.40$ cm$^2$.

**31.** **1st problem:** See 2nd problem, Q.**31.**, 2012 on page 45.

*Or*

**2nd problem:** See 2nd problem, Q.**30.**, 2010 on page 26.

**32.** Let $f_1$ and $f_2$ be the missing frequencies for the classes $20-40$ and $60-80$ respectively. Then we modify the given data as follows:

| Class | Mid point $(x_i)$ | Frequency $(f_i)$ | $f_i x_i$ |
|---|---|---|---|
| $0-20$ | 10 | 17 | 170 |
| $20-40$ | 30 | $f_1$ | $30f_1$ |
| $40-60$ | 50 | 32 | 1600 |
| $60-80$ | 70 | $f_2$ | $70f_2$ |
| $80-100$ | 90 | 19 | 1710 |
| | | $N = 68 + f_1 + f_2$ | $3480 + 30f_1 + 70f_2$ |

Now, we have

$$N = 58 + f_1 + f_2 \implies 120 - 68 = f_1 + f_2 \implies f_1 = 52 - f_2. \quad (1)$$

But mean $(\overline{x}) = 50$.

$$\begin{aligned}
&\therefore \frac{1}{N}\sum_{i=1}^{5} f_i x_i = 50 \\
&\implies \frac{1}{120}(3480 + 30f_1 + 70f_2) = 50 \\
&\implies 30f_1 + 70f_2 = 2520 \\
&\implies 30(52 - f_2) + 70f_2 = 2520 \quad (\because \text{by } (1)) \\
&\implies 40f_2 = 960 \\
&\implies f_2 = 24.
\end{aligned}$$

$\therefore\ f_1 = 52 - f_2 = 52 - 24 = 28$. Hence, the missing frequencies for the intervals $20-40$ and $60-80$ are 28 and 24 respectively.

# Chapter 2

# Practice Questions

Questions asked in the HSLC Exam (from 2010 to 2017) are not included in the following lists.

## Number System

1. Prove that one of every three consecutive integers is divisible by 3.
2. If $a$ is an odd integer, show that $a^2 + (a+2)^2 + (a+4)^2 + 1$ is divisible by 12.
3. Show that the product of any three consecutive even integers is divisible by 48.
4. Use Euclid's division lemma to show that the square of any positive integer cannot be of the from $5m + 2$ or $5m + 3$ for some integer $m$.
5. Prove that if $x$ and $y$ are both odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.
6. If $a = 5q + 2$ for some integer $q$, what is the HCF of $a$ and 5?
(Answer: 1. If $a = bq + r$, then $\gcd(a, b) = \gcd(b, r)$.)
7. Use Euclid's algorithm to find the HCF of 45 and 220.
8. Let $a$ and $b$ be two positive integers where $a > b$. Prove that the last divisor (or the last non-zero remainder) in the Euclid's algorithm for $a$ and $b$ is the HCF of $a$ and $b$.
9. State the fundamental theorem of arithmetic.
10. Give the canonical decomposition of an integer $n > 1$.

11. Explain why $11 \times 13 \times 19 + 7 \times 13$ is a composite number.

12. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons. (Answer: No. The HCF must divide the LCM.)

13. The product of the HCF and the LCM of two integers is equal to the product of the integers. Show that the above statement holds for the integers 168 and 780.

14. Show that one and only one out of $n$, $n+4$, $n+8$, $n+12$ and $n+16$ is divisible by 5, where $n$ is any positive integer.

15. Use fundamental theorem of arithmetic to show that any square number cannot be put in the form $4k+2$.

16. Find the LCM and the HCF of 56 and 120 by the prime factorisation method.

17. Can the number $6^n$, $n$ being a natural number, end with the digit 5? Give reasons. (Answer: No.)

18. Find any five consecutive composite numbers.

19. The LCM and the HCF of two numbers are 2079 and 27 respectively. If one number is 297, find the other number. (Answer: 189.)

20. Find the largest number which divides 318 and 739 leaving remainders 3 and 4 respectively. (Answer: 105.)

21. Find the least number greater than 7 which when divided by 24, 36 and 60 will leave in each case the same remainder 7. (Answer: 367.)

22. Find the least positive multiple of 13 which when divided by 6, 15 and 24 leaves the same remainder in each case. (Answer: 364.)

23. By what numbers may 1011 may be divided so that the remainder is 10? (Answer: 11, 13, 77, 91, 143, 1001.)

24. If $x, y, z \in \mathbb{R}, x \neq 0$ and $xy = xz$, prove that $y = z$.

25. Prove that for any $x, y \in \mathbb{R}$, $x(-y) = -xy$.

26. Prove that for any $x, y \in \mathbb{R}$, $(-x)(-y) = xy$.

27. For any $x \in \mathbb{R}$, prove that $|x| \geq 0$.

28. For any $x, y \in \mathbb{R}$, prove that $|xy| = |x||y|$.

29. For any $x, y \in \mathbb{R}$, prove that $|x+y| \leq |x| + |y|$.
30. For any $x, y \in \mathbb{R}$, prove that $|x-y| \geq ||x| - |y||$.
31. If $a^2 + b^2 = 0$, prove that $a = 0$ and $b = 0$.
32. Identify on the number line, the points $x$ satisfying $|x-2| \leq 3$.

## Polynomials

1. State the division algorithm for polynomials.
2. Divide $2x - x^3 + 6$ by $3x + 2 - x^2$ and verify the division algorithm.
3. When a polynomial $p(x)$ is divided by $x^2 - 2x - 2$, the quotient and remainder are $3x^2 + x - 3$ and $-3x - 7$ respectively. Find $p(x)$. (Answer: $3x^4 - 5x^3 - 11x^2 + x - 1$.)
4. On dividing $x^5 + 3x^2 + 9$ by a polynomial $f(x)$, the quotient and remainder are $x^3 - 4x^2 + 15x - 53$ and $197x + 62$ respectively. Find $f(x)$. (Answer: $f(x) = x^2 + 4x + 1$.)
5. Find, without actual division, the remainder when $3x^3 + 4x^2 - 5$ is divided by $x + 2$. (Answer: $-13$.)
6. Use factor theorem to determine whether $x+3$ is a factor of $x^3+2x^2-3$.
7. Find the value of $k$ if $3x + 2$ is a factor of $x^3 + 2x^2 + kx + 1$. (Answer: $43/18$.)
8. If $2x^2 + 5x + k$ and $x^2 + 3x + l$ are both divisible by $x + a$, show that $a = 2l - k$.
9. Factorise $2x^4 - 3x^2 + 6x + 7$ by using factor theorem.
10. Find the value of $k$ if $x^3 + y^3 + z^3 + kxyz$ is divisible by $x + y + z$.

    (Hint: Put $z = -x - y$. We have $x^3 + y^3 + (-x-y)^3 + kxy(-x-y) = 0$, by factor theorem. $-xy(x+y)(3+k) = 0$. Hence, $k = -3$.)
11. Factorise $x^2(y+z) + y^2(z+x) + z^2(x+y) + 2xyz$ by using factor theorem.
12. Factorise $x^2y^2(x^2 - y^2) + y^2z^2(y^2 - z^2) + z^2x^2(z^2 - x^2)$ by using factor theorem. (Answer: $-(x+y)(y+z)(z+x)(x-y)(y-z)(z-x)$.)
13. Prove that $x^n + a^n$ is divisible by $x + a$ if and only if $n$ is odd.
14. Prove that $x^n - a^n$ is divisible by $x + a$ if and only if $n$ is even.

## Factorisation

1. Is $x(y+z)+y(z+x)+z(x+y)$ a cyclic expression? Give reason.

2. Prove the following:

   **(a)** $a^2(b+c)+b^2(c+a)+c^2(a+b)+3abc=(a+b+c)(ab+bc+ca)$.

   **(b)** $a^2(b-c)+b^2(c-a)+c^2(a-b)=-(a-b)(b-c)(c-a)$.

   **(c)** $a^3(b^2-c^2)+b^3(c^2-a^2)+c^3(a^2-b^2)$
   $=-(a-b)(b-c)(c-a)(ab+bc+ca)$.

   **(d)** $(a+b+c)^3-a^3-b^3-c^3=3(a+b)(b+c)(c+a)$.

   **(e)** $(a+b+c)(bc+ca+ab)-abc=(a+b)(b+c)(c+a)$.

3. Factorise the following:

   **(i)** $8x^3+27y^3+64z-72xyz$,

   **(ii)** $a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)+3abc$,

   **(iii)** $x^2y^2(x-y)+y^2z^2(y-z)+z^2x^2(z-x)$,

   **(iv)** $8(a+b+c)^3-(b+c)^3-(c+a)^3-(a+b)^3$,

   **(v)** $a^2(b-c)+b^2(c-a)+c^2(a-b)$.

4. If $a^3+b^3+c^3=3abc$, prove that either $a+b+c=0$ or $a=b=c$.

5. Find the value of $x^2(y+z)+y^2(z+x)+z^2(x+y)+3xyz$ if $x+y+z=23$ and $x^2+y^2+z^2=181$. (Answer: 4002.)

6. If $x+y+z=9$, $xy+yz+zx=26$ and $xyz=24$, find the value of $x^2(y+z)+y^2(z+x)+z^2(x+y)$. (Answer: 162.)

## Pair of Linear Equations in Two Variables

1. When is a pair of linear equations said to be a consistent pair?

2. When is a pair of linear equations said to be an inconsistent pair?

3. State the condition under which the system of equations $ax+by+c=0$, $a_1x+b_1y+c_1=0$ has a unique solution.

4. State the condition under which the system of equations $ax+by+c=0$, $a_1x+b_1y+c_1=0$ has no solution.

5. State the condition under which the system of equations $ax+by+c=0$, $a_1x+b_1y+c_1=0$ has infinitely many solutions.

6. Examine whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident.

   **(i)** $4x - 3y = 5$, $3x - 5y = 1$, (Answer: Intersect at a point.)

   **(ii)** $x + 2y = 5$, $3x + 6y = 7$, (Answer: Parallel.)

   **(iii)** $5x + 2y = 4$, $10x + 4y = 8$. (Answer: Coincident.)

7. Prove the validity of the cross-multiplication method for solving a consistent pair of linear equations.

8. Solve the following pair of linear equations by the substitution method:

   $$x + 2y = 7, \quad x - 3y + 3 = 0.$$ (Answer: $x = 3$, $y = 2$.)

9. Solve the following pair of linear equations by the elimination method:

   $$2x + y = 3, \quad 3x + 4y = 2.$$ (Answer: $x = 2$, $y = -1$.)

10. Solve the following pair of linear equations by cross-multiplication method:

    $$4x + 3y = 12, \quad 5x - 3y = 15.$$ (Answer: $x = 3$, $y = 0$.)

    Give the geometrical interpretation of the solution.

11. Solve the following pair of equations:

    $$\frac{2xy}{x+y} = \frac{3}{2}, \quad \frac{xy}{2x-y} = -\frac{3}{10},$$

    where $x + y \neq 0$, $2x - y \neq 0$. (Answer: $x = 1/2$, $y = -3/2$.)

12. Find the conditions for which the system of equations

    $$2x + 3y = 7, \qquad (p+q)x + (2p-q)y = 21$$

    is **(i)** dependent, **(ii)** consistent, **(iii)** inconsistent.

    (Answers: (i) $p = 5$, $q = 1$, (ii) $p \neq 5q$, (iii) $p = 5q$ but $p \neq 5$, $q \neq 1$.)

13. Solve graphically the following system of linear equations:

    $$x + y = 1, \qquad 3x + 2y = 1.$$ (Answer: $x = -1$, $y = 2$.)

14. Four years ago a father was nine times as old as his son, and 8 years hence the father's age will be three times the son's age. Find their present ages. (Answer: 40 years, 8 years.)

15. A train covered a certain distance at a uniform speed. If the train had been 10 km/hr faster, it would have taken 6 hours less than the scheduled time. And, if the train were slower by 10 km/hr, it would have taken 9 hours more than the scheduled time. Find the length of the journey. (Answer: 1800 km.)

16. Tombi can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current. (Answer: 6 km/hr and 4 km/hr.)

17. A chair and a table cost ₹ 1,200. By selling the chair at a profit of 20% and the table at a loss of 5%, there is a profit of 4% as a whole. Find the cost price of the chair and that of the table. (Answer: ₹ 432, ₹ 768.)

## Quadratic Equations

1. Write the standard form of a quadratic equation.

2. Derive the quadratic formula.

3. Solve $12x^2 - 7x + 1 = 0$ by the method of factorisation and verify the result. (Answer: 1/3, 1/4.)

4. Solve $3x^2 - 16x + 5 = 0$ by the method of completing the perfect square and verify the result. (Answer: 1/3, 5.)

5. Solve $2x^2 - 34 - 336 = 0$ by using the quadratic formula. (Answer: $-7$, 24.)

6. The nature of the roots of a quadratic equation depends on the value of its discriminant. Justify.

7. If one root of the quadratic equation $2x^2 + kx - 6 = 0$ is 2, find the value of $k$. Also, find the other root. (Answer: $k = -1$.)

8. When is the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, said to have real roots?

9. Without solving, determine the nature of roots of the quadratic equation $2x^2 + x - 3 = 0$. (Answer: Real and unequal.)

10. If $\alpha$ and $\beta$ are the roots of the equation $x^2 + px - q = 0$, find the values of $\alpha^3 + b^3$ and $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. (Answer: $-p^3 - 3pq$, $\frac{p^2+2q}{q^2}$.)

11. If $\alpha$ and $\beta$ are the roots of $ax^2 + bx + c = 0$, find the equation whose roots are $\alpha + 2$ and $\beta + 2$. (Answer: $ax^2 - (4a - b)x + c - 2b + 4a = 0$.)

12. If $\alpha$ and $\beta$ are the roots of $2x^2 - 5x - 4 = 0$, find a quadratic equation whose roots are $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$. (Answer: $8x^2 - 10x - 61 = 0$.)

13. Find the values of $a$ for which one of the roots of $x^2+(2a+1)x+(a^2+2) = 0$ is twice the other root. Find also the roots of this equation for these values of $a$. (Answer: $a = 4$.)

14. If the roots of $ax^2 + bx + c = 0$ are in the ratio $m : n$, show that $(b^2 - 4ac)mn = ac(m - n)^2$.

15. If $\alpha$, $\beta$ are the roots of the equation $x^2 + px + q = 0$, $q \neq 0$, then find the values of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ in terms of $p$ and $q$. (Answer: $\frac{p^2-2q}{q^2}$.)

16. Find the value of $p$ so that the equation $3px^2 - 12x + p = 0$ has equal roots. (Answer: $\pm 2\sqrt{3}$.)

17. Leishna's mother is 26 years older than her. The product of their ages (in years) 3 years from now will be 360. Find the present age of Leishna. (Answer: 7 years.)

18. Two sides of a right-angled triangle are $a$ cm and $a + 1$ cm. If the length of the hypotenuse is 29 cm, find the area of the right-angled triangle. (Answer: 210 $cm^2$.)

19. The sum of two numbers is 7 and the difference of their reciprocals is $\frac{1}{12}$. What are the two numbers? (Answer: (3 and 4) or (28 and -21).)

20. A number consists two digits. The digit in the units place is the square of the digit in tens place. If the sum of the digits is 12, find the number. (Answer: 39.)

21. The sides containing the right angle of a right triangle differ in length by 7 cm. If the hypotenuse is 13 cm long, determine the lengths of the two sides of the triangle. (Answer: 5 cm, 12 cm.)

22. A rectangular looking glass 18 cm by 12 cm has a wooden frame of uniform width. If the area of the frame is equal to that of the looking glass, find the dimensions of the framed glass. (Answer: 24 cm $\times$ 18 cm.)

# Arithmetic Progression (AP)

1. Define finite and infinite sequences.

2. Can the common difference of an AP be zero? (Answer: Yes.)

3. In an AP, if the third term is four times the first term, and the sixth term is 17, find the first term and the common difference of the AP. (Answer: First term = 2. Common difference = 3.)

4. The $n^{\text{th}}$ term of a sequence is given by $a_n = 5n + 2$. Show that the sequence is an AP.

5. Is 0 a term of the AP: $31, 28, 25, \ldots$? Give reason. (Answer: No.)

6. If the $m^{\text{th}}$ term of an AP is $\dfrac{1}{n}$ and the $n^{\text{th}}$ term is $\dfrac{1}{m}$, then show that the $(mn)^{\text{th}}$ term of the AP is 1.

7. If $a$, $b$ and $c$ are the $p^{\text{th}}$, $q^{\text{th}}$ and $r^{\text{th}}$ terms of an AP respectively, prove that $a(q - r) + b(r - p) + c(p - q) = 0$.

8. If the $p^{\text{th}}$ term of an AP is $q$ and the $q^{\text{th}}$ term is $p$, prove that its $n^{\text{th}}$ is $p + q - n$.

9. If $m$ times the $m^{\text{th}}$ term of an AP is equal to $n$ times the $n^{\text{th}}$ term, show that the $(m + n)^{\text{th}}$ term of the AP is zero.

10. The first term of an AP is 28 and the common difference is $-4$. How many terms of the AP must be taken so that their sum is 100? Explain the double answer. (Answer: 5 or 10.)

11. The sum of three numbers in AP is 12 and the sum of their cubes is 408. Find the numbers. (Answer: $1, 4, 7$.)

12. Find the $15^{\text{th}}$ term from the last term (towards the first term) of the AP: $3, 7, 11, \ldots, 123$. (Answer: 67.)

13. Show that there are 30 two-digit numbers divisible by 3.

14. In an auditorium, the seats are so arranged that there are 8 seats in the first row, 11 seats in the second row, 14 seats in the third row, etc., thereby increasing the number of seats by 3 every next row. If there are 50 seats in the last row, how many rows of seats are there in the auditorium? (Answer: 15.)

15. The sums of first $n$ terms of two APs are in the ratio $7n+1 : 4n+27$. Find the ratio of their $11^{\text{th}}$ terms. (Answer: $4:3$.)

16. The sums of the first $p, q, r$ terms of an AP are $a, b, c$ respectively. Show that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$.

17. Find the first term and the common difference of an AP if the sum of the first $n$ terms is $\dfrac{n(5n+7)}{12}$.
(Answer: First term = 1. Common difference = 5/6.)

18. In an AP, if the sum of the first $m$ terms is equal to $n$ and that of the first $n$ terms is equal to $m$, then prove that the sum of the first $(m+n)$ terms of the AP is $-(m+n)$.

19. The sums of the first $n$, $2n$, $3n$ terms of an AP are $S_1$, $S_2$, $S_3$ respectively. Show that $S_3 = 3(S_2 - S_1)$.

## Triangles

1. When are two polygons said to be similar?

2. When are two triangles said to be similar?

3. State and prove the converse of basic proportionality theorem.

4. In a $\triangle ABC$, $D$ and $E$ are points on the sides $AB$ and $AC$ respectively such that $AD \times EC = AE \times DB$. Prove that $DE \| BC$.

5. If the medians of a triangle are the bisectors of the corresponding angles of the triangle, prove that the triangle is equilateral.

6. State AA similarity criterion.

7. It is given that $\triangle FED \sim \triangle STU$. Is it true that $\dfrac{DE}{ST}=\dfrac{EF}{TU}$? Why?
(Answer: No. $F \leftrightarrow S$, $E \leftrightarrow T$, $D \leftrightarrow U$. So, $\frac{EF}{ST}=\frac{DE}{TU}$.)

8. Is it true to say that if in two triangles, an angle of one triangle is equal to an angle of another triangle and two sides of one triangle are proportional to the two sides of the other triangle, then the triangles are similar? Give reasons for your answer. (Answer: No.)

9. Two vertical poles of heights $a$ metres and $b$ metres are $p$ metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the other is $\dfrac{ab}{a+b}$ metres.

10. If two triangles are similar, prove that the corresponding medians are proportional.

11. $ABC$ is an isosceles triangle in which $AB = AC$ and $D$ is a point on $AC$ such that $BC^2 = AC \cdot CD$. Prove that $BC = BD$.

12. If one angle of a triangle is equal to one angle of another triangle and the bisectors of these equal angles divide the opposite sides in the same ratio, prove that the triangles are similar.

13. Prove that the areas of two similar triangles are in the ratio of the square of the corresponding medians.

14. If $D$, $E$, $F$ are respectively the mid-points of the sides $BC$, $CA$, $AB$ of a $\triangle ABC$, prove that $\dfrac{\text{ar}(\triangle DEF)}{\text{ar}(\triangle ABC)} = \dfrac{1}{4}$.

15. A ladder 13 m long reaches a window which is 12 m above the ground on one side of a street. Keeping its foot at the same point, the ladder is turned to the other side of the street and it just reaches a window of 5 m high. Find the width of the street. (Answer: 17 m.)

16. If $D$, $E$, $F$ are the mid-points of the sides $BC$, $CA$, $AB$ of a right $\triangle ABC$, right angled at $A$, respectively, prove that $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$.

17. Find the length of the altitude and the area of an equilateral triangle having $a$ as the length of a side.

18. In a right triangle $ABC$ right angled at $C$, if $p$ is the length of the perpendicular segment drawn from $C$ upon $AB$, prove that

    **(i)** $ab = pc$, **(ii)** $\dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{p^2}$, where $a = BC$, $b = CA$ and $c = AB$.

19. If $A$ is the area of a right triangle and $b$ one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\dfrac{2Ab}{\sqrt{b^4 + 4A^2}}$.

20. Prove that the sum of the squares on four sides of a rhombus is equal to the sum of the squares on the diagonals.

21. $D$ and $E$ are the midpoints of the sides $AB$ and $AC$ respectively of a right $\triangle ABC$ right angled at $A$. Prove that $4(BE^2 + CD^2) = 5BC^2$.

## Circles

1. How many tangents can be drawn to a circle through a point inside the circle? (Answer: Zero.)

2. $OP$ is a radius of a circle with centre $O$. Prove that the line drawn through $P$, perpendicular to $OP$ is the tangent to the circle at $P$.

3. The radius of the incircle of a $\triangle ABC$ is 4 cm. The incircle touches the side $BC$ of the $\triangle ABC$ at the point $D$. If $BD = 8$ cm and $DC = 6$ cm, find $AB$ and $AC$. (Answer: $AB = 15$ cm, $AC = 13$ cm.)

4. $PA$ and $PB$ are the tangent segments drawn from an external point $P$ to a circle with centre $O$. If $\angle AOP = 70°$, find at what angle the two tangents are inclined to each other. (Answer: 110°.)

5. If $PA$ and $PB$ are the tangent segments drawn from an external point $P$ to a circle whose centre is $O$, prove that $OP$ bisects $AB$ and $OP \perp AB$.

6. Two concentric circles are of radii 5 cm and 13 cm. Find the length of the chord of the larger circle which touches the smaller circle. (Answer: 24 cm.)

7. Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

8. $\triangle ABC$ is isosceles with $AB = AC$. The incircle of the $\triangle ABC$ touches $BC$ at $P$. Prove that $BP = CP$.

9. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

## Construction

1. Draw any line segment and divide it internally in the ratio $2 : 5$. Write the steps of construction.

2. Draw a line segment $PQ = 10$ cm. Take a point $A$ on $PQ$ such that $PA : PQ = 2 : 5$. Measure the lengths of $PA$ and $AQ$.

3. To divide a line segment $AB$ in the ratio 5 : 7, a ray $AX$ is drawn such that $\angle BAX$ is an acute angle, then a ray $BY$ is drawn parallel to $AX$ and the points $A_1, A_2, A_3, \ldots$ and $B_1, B_2, B_3, \ldots$ are located at equal distances on the rays $AX$ and $BY$, respectively. Which points are joined in the next step? (Answer: $A_5$ and $B_7$.)

4. What do you mean by 'scale factor' in the construction of a triangle similar to a given triangle?

5. To construct a triangle similar to a given $\triangle ABC$ with its sides $\frac{5}{11}$ of the corresponding sides of $\triangle ABC$, first a ray $BX$ is drawn such that $\angle CBX$ is an acute angle and $X$ lies on the side opposite to the vertex $A$ with respect to $BC$. Then the points $Q_1, Q_2, Q_3, \ldots$ are located on the ray $BX$ at equal distances. Which points are joined in the next step? (Answer: $Q_{11}$ and $C$.)

6. Construct a triangle similar to a given triangle $ABC$ with its sides equal to $\frac{4}{7}$ of the corresponding sides of the triangle $ABC$. Write the steps of construction.

7. Construct a triangle similar to a given triangle $ABC$ with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle $ABC$. Write the steps of construction.

8. Construct a pair of tangents to a circle, inclined to each other at $60°$. Write the steps of construction.

## Trigonometry

1. Define the sine of an acute angle.

2. If $\sin\theta = \dfrac{a^2 - b^2}{a^2 + b^2}$, find the value of the other trigonometric ratios of $\theta$.

3. Prove that the values of trigonometric ratios of an acute angle are uniquely defined and do no vary with size of the right triangle considered.

4. Show that $0 < \sin\theta < 1$ for any acute angle $\theta$.

5. Show that for each positive real number $x$, there exists an acute angle $\theta$ such that $\tan\theta = x$.

6. Find the values of the trigonometric ratios of $30°$.

7. If $\sin(A+B) = \frac{\sqrt{3}}{2}$, $\cos(A-B) = \frac{\sqrt{3}}{2}$, $0° < A+B \leq 90°$, $A > B$, find $A$ and $B$. (Answer: $A = 45°$, $B = 15°$.)

8. Define a trigonometric identity.

9. Show that $\sin A = \frac{1}{\operatorname{cosec} A}$ for any acute angle $A$.

10. Prove that $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$, where $0 < \theta \leq 90$.

11. Prove that the cosine of an acute angle is equal to the sine of its complementary angle and vice versa.

12. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

    **(a)** $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$,

    **(b)** $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$,

    **(c)** $(\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = (1 + \sec\theta\operatorname{cosec}\theta)^2$,

    **(d)** $\tan^2\theta - \tan^2\phi = \frac{\sin^2\theta - \sin^2\phi}{\cos^2\theta\cos^2\phi}$,

    **(e)** $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$.

13. If $\tan\theta = \frac{4}{3}$, show that $\frac{4\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta} = \frac{7}{25}$.

14. If $\sin\theta = \cos(\theta + 6°)$, where $2\theta$ and $\theta + 6°$ are acute angles, find the value of $\theta$. (Answer: $\theta = 28°$.)

15. If $A$, $B$, $C$ are the interior angles of a triangle $ABC$, prove the following:

    **(a)** $\cot\frac{B+C}{2} = \tan\frac{A}{2}$,

    **(b)** $\cos\frac{B+C}{2}\cos\frac{C+A}{2}\cos\frac{A+B}{2} = \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$.

16. Define the angle of elevation and the angle of depression.

17. The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45°, respectively. Find the height of the multi-storeyed building and the distance between the two buildings.
    (Answer: $4(3+\sqrt{3})$ m, $4(3+\sqrt{3})$ m.)

18. If the angle of elevation of a cloud from a point $h$ metres above a lake is $\alpha$ and the angle of depression of its reflection in the lake is $\beta$, prove that the height of the cloud is $\dfrac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$.

19. The angles of elevation of the top of a tower from two points $d$ metres apart on a horizontal line from the foot of the tower are $\alpha$ and $\beta$.

    **(a)** If the two points are on the same side of the tower and $\alpha > \beta$, prove that the height of the tower is $\dfrac{d\tan\alpha\cdot\tan\beta}{\tan\alpha - \tan\beta}$.

    **(b)** If the two points are on the opposite sides of the tower, prove that the height of the tower is $\dfrac{d\tan\alpha\cdot\tan\beta}{\tan\alpha + \tan\beta}$.

## Coordinate Geometry

1. Find the coordinates of the centroid of the triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. (Answer: $(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3})$.)

2. The mid point of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $M(1, 2a+1)$. Find the values of $a$ and $b$. (Answer: $a = 2$, $b = 2$.)

3. $AB$ is a diameter of a circle with centre $C(-1, 6)$. If the coordinates of $A$ are $(-7, 3)$, find the coordinates of $B$. (Answer: $(5, 9)$.)

4. Find the ratio in which the line segment joining $(-2, -3)$ and $(5, 6)$ is divided by $x$-axis. Also, find the coordinates of the point of division. (Answer: $1 : 2$, $(1/3, 0)$.)

5. $ABCD$ is a parallelogram with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x, y)$ taken in order. Find the values of $x$ and $y$ in terms of $x_1$, $x_2$, $x_3$, $y_1$, $y_2$ and $y_3$. (Answer: $x = x_1 + x_3 - x_2$, $y = y_1 + y_3 - y_2$.)

6. If the points $A(a, -11)$, $B(5, b)$, $C(2, 15)$ and $D(1, 1)$ taken in order are the vertices of a parallelogram $ABCD$, find the values of $a$ and $b$. (Answer: $a = 4$, $b = 3$.)

7. Prove that the points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear if $\dfrac{1}{a} + \dfrac{1}{b} = 1$.

8. If $a$, $b$, $c$ are distinct, prove that the points $(a, a^2)$, $(b, b^2)$, $(c, c^2)$ can never be collinear.

9. If the points $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ lie on the same line, prove that

$$\frac{y_2 - y_3}{x_2 x_3} + \frac{y_3 - y_1}{x_3 x_1} + \frac{y_1 - y_2}{x_1 x_2} = 0, \text{ provided } x_1 x_2 x_3 \neq 0.$$

10. Using coordinate geometry, prove that the diagonals of a parallelogram divide the parallelogram into four triangles of equal areas.

11. Show that the area of the parallelogram whose vertices, taken in order, are $(0, 0)$, $(a, b)$, $(c, d)$, $(a + c, b + d)$ is $|ad - bc|$ sq. units.

## Mensuration

1. Find the radius and the area of the circle whose circumference is equal to the sum of the circumferences of two circles of radius 19 cm and 9 cm respectively. (Answer: 28 cm, 2464 cm$^2$.)

2. The diameter of the driving wheel of a bus is 140 cm. How many revolutions must the wheel make in 11 minutes in order to keep a speed of 60 km per hour? (Answer: 2500.)

3. The circumference of a circle exceeds the diameter by 21 cm. Find the area of the circle. (Answer: 75.46 cm$^2$.)

4. Find the area of the largest circle that can be drawn inside a square of side 28 cm. (Answer: 616 cm$^2$.)

5. The cost of levelling a path of uniform width which surrounds a circular park of diameter 320 m at the rate of ₹ 7.00 per square metre is ₹ 35750. Find the width of the path. (Answer: 5 m.)

6. The area of a segment of a circle is equal to the area of the corresponding sector minus the area of the corresponding triangle. Do you agree with this statement? Give reason for your answer. (Answer: No.)

7. Find the sectorial angle and the area of the sector of a circle with radius 36 cm if the length of the corresponding arc is $3\pi$ cm.
(Answer: 15°, $54\pi$ cm$^2$.)

8. The length of the minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time period 6:05 am and 6:40 am.
(Answer: 45.83 cm$^2$.)

9. A chord of a circle of radius 12 cm subtends 120° at the centre. Find the area of the corresponding minor segment of the circle.
(Answer: $4(4\pi - 3\sqrt{3})$ cm$^2$.)

10. A chord of a circle subtends an angle of 60° at the centre. If the length of the chord is 12 cm, find the areas of the two segments into which the chord divides the circle. Take $\pi = 3.14$ and $\sqrt{3} = 1.73$.
(Answer: 13.08 cm$^2$, 439.08 cm$^2$.)

11. A circle is inscribed in an equilateral triangle of sides 18 cm. Find the area between the triangle and the circle. Take $\sqrt{3} = 1.73$.
(Answer: 55.47 cm$^2$.)

12. From an equilateral triangle of sides 14 cm, a sector of radius 7 cm with centre at a vertex and enclosed by two sides, is cut off. Find the area of the remaining portion. Take $\sqrt{3} = 1.73$. (Answer: 22.4 cm$^2$.)

13. A solid is in the form of a cylinder of diameter 7 cm with hemispherical ends. If the total length of the solid is 42 cm, find the total surface area and the volume of the solid. (Answer: 924 cm$^2$, 1527.16 cm$^3$.)

14. From a solid cylinder of height 28 cm and radius 12 cm, a cone of the same height and same radius is removed. Find the volume of the remaining solid. (Answer: 8448 cm$^3$.)

15. A solid is in the form of a cylinder surmounted by a cone of the same radius. If the radius of the base and the height of the cone are '$r$' and '$h$' respectively and the total height of the solid is $3h$, prove that the volume of the solid is $\frac{7}{3}\pi hr^2$.

16. A solid metallic cone is 81 cm high and the radius of its base is 6 cm. If it is melted and recast into a solid sphere, find the curved surface area of the sphere. (Answer: 1018.29 cm$^2$.)

17. A solid metallic cone is melted and recast into a solid cylinder of the same height. Prove that the radii of the cone and the cylinder are in the ratio $\sqrt{3} : 1$.

18. A solid metallic cylinder and another solid metallic cone have the same height $h$ and the same radius $r$. If the two solids are melted together and recast into a cylinder of radius $\frac{1}{2}r$, prove that the height of the new cylinder is $\frac{16}{3}h$.

19. The radii of the bases of two solid metallic cones of same height $h$ are $x_1$ and $x_2$. If the two cones are melted together and recast into a cylinder of height $h$, then show that the radius of the base of the cylinder is $\sqrt{\frac{1}{3}\left(x_1^2 + x_2^2\right)}$.

20. The rain water from a roof of dimensions 22 m × 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the rain water collected from the roof just fill the cylindrical vessel, then find the rainfall in cm. (Answer: 2.5 cm.)

21. Derive the formulas for finding the volume and the total surface area of a frustum of a right circular cone.

22. From a cone of height 24 cm, a frustum is cut off by a plane parallel to the base of the cone. If the volume of the frustum is $\frac{19}{27}$ of the volume of the cone, find the height of the frustum. (Answer: 8 cm.)

23. A cone is divided into two parts by a plane through the mid-point of the axis of the cone and parallel to the base. Show that the ratio of the volume of the conical part to that of the frustum is 1 : 7.

## Statistics

1. Which measure of central tendency is represented by the abscissa of the point where the less than ogive and the more than ogive intersect? (Answer: Median.)

2. The mean of an ungrouped data and the mean calculated when the same data is grouped are always the same. Do you agree with this statement? Give reason for your answer. (Answer: No.)

3. Is it true to say that the mean, mode and median of grouped data will always be different? Justify your answer. (Answer: No.)

4. If the mean and the median of a frequency distribution differs by 1.05, estimate the difference between the mean and the mode using Pearson's empirical formula. (Answer: 3.15.)

5. Establish the formula: $\overline{x} = a + \frac{1}{N}\sum f_i d_i$ for finding the mean of a frequency distribution, where the symbols have their usual meanings.

6. Establish the formula: $\overline{x} = a + h\overline{u}$ for finding the mean of a frequency distribution, where the symbols have their usual meanings.

7. The following table shows the daily income of 50 workers of a factory.

| Daily income (in ₹) | 100 – 120 | 120 – 140 | 140 – 160 | 160 – 180 | 180 – 200 |
|---|---|---|---|---|---|
| Number of workers | 12 | 14 | 8 | 6 | 10 |

Find the mean, the median and the mode of the above data. Compare and interpret the three measures of central tendency.

(Answer: Mean = ₹ 145.20. Median = ₹ 138.57. Mode = ₹ 125.00. The average daily income of a worker is ₹ 145.20. The daily income of about half the workers is less than ₹ 138.57 and the daily income of the other half is more than ₹ 138.57. The daily income of the maximum number of workers is around ₹ 125.00.)

8. Find the median of the following data. (Answer: 145.14.)

| Class | 118 – 126 | 127 – 135 | 136 – 144 | 145 – 153 | 154 – 162 |
|---|---|---|---|---|---|
| Frequency | 3 | 6 | 10 | 14 | 7 |

9. The mean of the following frequency distribution is 57.6 and the sum of the observation is 50. Find the missing frequencies : $f_1$ and $f_2$.

| Class | 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 | 100 – 120 |
|---|---|---|---|---|---|---|
| Frequency | 7 | $f_1$ | 12 | $f_2$ | 8 | 5 |

(Answer: $f_1 = 8$, $f_2 = 10$.)

10. The median of the following frequency distribution is 78 and the sum of the observation is 50. Find the missing frequencies : $f_1$ and $f_2$.

| Class | 64–68 | 68–72 | 72–76 | 76–80 | 80–84 | 84–88 | 88–92 |
|---|---|---|---|---|---|---|---|
| Frequency | 4 | 5 | $f_1$ | $f_2$ | 9 | 7 | 1 |

(Answer: $f_1 = 8$, $f_2 = 16$.)

11. Find the missing frequency '$x$' in the following frequency distribution if the mode is 67. (Answer: $x = 8$.)

| Class | 40 – 50 | 50 – 60 | 60 – 70 | 70 – 80 | 80 – 90 |
|---|---|---|---|---|---|
| Frequency | 5 | $x$ | 15 | 12 | 7 |

12. A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the following data was obtained.

| Height (in cm) | Number of girls |
|---|---|
| Less than 140 | 4 |
| Less than 145 | 11 |
| Less than 150 | 29 |
| Less than 155 | 40 |
| Less than 160 | 46 |
| Less than 165 | 51 |

Find the 4$^{\text{th}}$ decile and the 74$^{\text{th}}$ percentile of the above data. By drawing the less than ogive, indicate the positions of the quartiles.
(Answer: $D_4 = 147.61$. $P_{74} = 153.97$.)

13. Find the lower and the upper quartiles of the following distribution.

| Marks | Number of students |
|---|---|
| 0 and above | 80 |
| 10 and above | 77 |
| 20 and above | 72 |
| 30 and above | 65 |
| 40 and above | 55 |
| 50 and above | 43 |
| 60 and above | 28 |
| 70 and above | 16 |
| 80 and above | 10 |
| 90 and above | 8 |
| 100 and above | 0 |

By drawing the more than ogive, indicate the position of the median.
(Answer: $Q_1 = 35$. $Q_3 = 66.67$.)

14. The more than ogive curve and the less than ogive curve of a frequency distribution intersect each other at at the point $(40, 60)$. What is the median of the distribution? (Answer: 40.)

15. Which measure of central tendency will be the most suitable in each of the following cases? Justify your answer in each case.

    **(a)** To determine the productivity of a field using the data of the yield of the field for the past twenty years.

    **(b)** To find the average wage in a country.

    **(c)** To find the most popular T.V. programme being watched.

(Answer: (a) Mean. (b) Median. (c) Mode.)

# Probability

1. Distinguish between subjective and objective probabilities giving examples in each case.

2. When do you say that a die is fair?

3. Write down the sample space of tossing a coin and rolling a die simultaneously.

4. When are two events said to be independent?

5. Are the two events $\{H\}$ and $\{T\}$ in tossing a coin independent? Justify your answer. (Answer: No.)

6. Define an elementary event.

7. Define an exhaustive set of events.

8. When are two events said to be complementary?

9. Define the terms **(i)** sure event and **(ii)** impossible event, associated with a random experiment.

10. $0 \leq P(A) \leq 1$ for any event $A$. Justify.

11. Write down the relationship between the empirical probability and the classical probability of an event.

12. If I toss a coin 3 times and get head each time, should I expect a tail to have a higher chance in the $4^{\text{th}}$ toss? Give reason in support of your answer. (Answer: No. Independent trials.)

13. In a situation that has only two possible outcomes, is it possible that the happening of an outcome has probability not equal to $\frac{1}{2}$? Give reasons. (Answer: Yes, if the outcomes are not equally likely.)

14. If you throw a die, it will show up 1 or 'not 1'. Therefore, the probability of getting 1 and the probability of getting 'not 1' each is equal to $\frac{1}{2}$. Is this correct? (Answer: No. Outcomes are not equally likely.)

15. Two fair dice are tossed. If the number on one die is $a$ and the number on the other die is $b$, what is the probability that $10a + b$ is a prime number? (Answer: 2/9.)

16. A letter is chosen from the word **'MATHEMATICS'**. Find the probability that the letter is **(i)** a vowel, **(ii)** a consonant.
(Answer: (i) 4/11, (ii) 7/11.)

17. Two cards are drawn from a pack of cards. Find the probability that the first card is a King and the second card is a Queen. (Answer: $\frac{4}{663}$.)

18. From an urn containing 3 white and 5 red balls, two balls are drawn at random. Find the probability that at least one is white. (Answer: $\frac{9}{14}$.)

19. Given that $p$ is the probability that a person aged $x$ years will die in a year, find the probability that none of the four persons all aged $x$ years will die in a year. (Answer: $(1-p)^4$.)

20. There are five men and four ladies in a council. If two council members are selected at random for a committee, how likely is it that both are ladies? (Answer: 1/6.)

21. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9. (Answer: 4/9.)

22. Two dice are thrown at random. Find the probability that the numbers shown are coprime. (Answer: 23/36.)

23. An integer is chosen between 0 and 100. What is the probability that the integer is
    **(a)** divisible by 7? (Answer: 14/99.)
    **(b)** not divisible by 7? (Answer: 85/99.)

24. A die has its six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and the total score is recorded.
    **(a)** How many different scores are possible? (Answer: Six.)
    **(b)** What is the probability of getting a total of 7? (Answer: 1/3.)

25. If a leap year is selected at random, what is the probability that it will have 53 Sundays? (Answer: 2/7.)

26. Considering suitable random experiments, give examples of each of the following:
    **(a)** Two events which are mutually exclusive but not independent.
    **(b)** Two events which are independent but not mutually exclusive.
    **(c)** Two events which are neither independent nor mutually exclusive.
    **(d)** Two events which are both independent and mutually exclusive.